Membrane Technology
ISSN (online): 1873-4049

Research on Key Nodes Identification in Intermodal Transportation Network Considering Node Strength

Jiaxin Zhang, Junxi Chen, Yan Zhang, Zhenlin Wei*
School of Traffic and Transportation, Beijing Jiaotong University, Beijing 100044, China
**Corresponding Author.*

Abstract:
Identifying key nodes in the coal rail-water intermodal transportation network is crucial for national energy supply, but traditional identification methods often overlook the critical factor of node strength. This paper introduced, for the first time, a Grey Relation Analysis based Importance Evaluation method considering Node Strength (GRAIE-NS), which combined local indicators of node degree with global indicators of node efficiency to achieve precise quantification of node importance. To verify the effectiveness of the GRAIE-NS method, an analysis was conducted using the actual operational network of coal rail-water intermodal transportation in China, comparing this method with classic key node identification approaches. Finally, a sensitivity analysis was performed to assess the impact of variations in the weights of initial importance and node strength on network invulnerability. The results indicate that the proposed method demonstrates high accuracy, particularly when the weight of initial importance is significant, as the failure of identified key nodes can have a substantial impact on the network.

Keywords: coal; rail-water intermodal transportation network; key nodes; node strength; invulnerability.

INTRODUCTION

Coal constitutes the primary energy source in China, representing 55.3% of the total energy consumption in 2023. Given the uneven distribution of coal resources, China addresses regional demand through strategies such as “West-to-East Coal Transport,” “North-to-South Coal Transport,” and “Coal Export through Passes.” Coal transportation primarily relies on railways (70%) and waterways (10%), establishing a pattern dominated by railways and supplemented by waterways. The coal rail-water intermodal system leverages the strengths of both railways and waterways, efficiently transporting coal to ports by rail and then delivering it to markets at a lower cost via water. This approach not only reduces transportation costs, energy consumption, and carbon emissions but also alleviates railway congestion and decreases road transportation volume[1]. Li and Zhao[2]further emphasize that rail-water intermodal transportation offers distinct advantages in cost reduction, enhanced transportation resource utilization, and meeting customer demands. Disruption to the coal rail-water intermodal network would significantly impact China's coal transportation framework. Therefore, the scientific identification and protection of critical nodes within this network are crucial for ensuring China's energy security and economic stability.

The remaining sections of the paper are structured as follows: In Sect. 2, we review the existing key node identification methods. In Sect. 3, we primarily focus on modeling the coal rail-water intermodal transportation network. In Sect. 4, we introduce the concept of node comprehensive importance and the node strength calculation model based on minimizing coal turnover volume for identifying key nodes in the coal rail-water intermodal transportation network from a multidimensional perspective. In Sect. 5, we present a case study on China's coal rail-water intermodal transportation network, using the comprehensive importance index to evaluate the key nodes and modeling four different attack scenarios. This section also explores and analyzes the impact of different initial importance values and node strength with varying weightings on the network's robustness after the failure of identified key nodes. In Sect. 6, we conclude with the main findings and future research directions.

LITERATURE REVIEW

Traditional methods for identifying key nodes primarily depend on the topological structure of the network[3-8], assessing their importance by calculating centrality metrics. Additionally, An et al.[9]proposed a method for identifying key nodes based on dynamic influence range and community importance, designed to address challenges related to node heterogeneity and inter-layer correlations in multilayer networks. Li et al.[10]introduced a method that integrates the local structural entropy and clustering coefficient of nodes to identify key nodes in complex networks, considering both the neighborhood influence of nodes and adjacency characteristics. Tian and Hu[11]developed a method using non-negative matrix factorization to identify key connecting nodes between overlapping communities in dynamic networks. Zhong et al.[12]proposed an information entropy-based hybrid influence method for identifying key nodes in complex networks. Zhang et al.[13]proposed a network representation learning and key node identification algorithm, which makes full use of network embedding techniques to extract node feature representations. Ren et al.[14]proposed an algorithm based on a local attraction model to mine key nodes in complex networks. Li et al.[15]generated feature embeddings for each node in the transportation network by combining dynamic adaptive attention networks and static feature embedding methods, and used a multi-layer perceptron for node classification to identify key nodes. Zhao et al.[16]evaluated centrality using information within a node's three-hop neighborhood, identifying key nodes in large-scale networks with low time complexity and high accuracy. Qu et al.[17]proposed a memetic algorithm that combines global search and local search to identify critical nodes in multiplex complex networks by considering multiple network metrics. Fu et al.[18]accurately identify critical nodes by calculating the relative gravity, direct gravity, and aggregating the influence of multi-layer neighbor nodes using information entropy. Lei et al.[19]proposed a method based on weighted fusion of information distance and weighted information index, which identifies critical nodes in the network by considering second-order neighboring nodes and their similarity.

Coal rail-water intermodal transportation, unlike single-mode transportation, involves the seamless integration of railways and waterways, featuring cross-regional and multimodal transportation. Furthermore, coal transportation imposes specific safety and connectivity requirements, resulting in heightened complexity and uniqueness in resource allocation, node connectivity, and network invulnerability. Traditional methods predominantly focus on local metrics such as node degree and clustering coefficient, failing to comprehensively account for global factors. Therefore, from the perspective of ensuring national energy security, this study proposes a method that integrates multi-dimensional key metrics to evaluate node importance, tailored to the characteristics of China's coal rail-water intermodal network. This method constructs an optimization model aiming to minimize coal turnover, combining local and global metrics such as node degree, node strength, and node efficiency to comprehensively identify key nodes and assess their impact on network performance, thereby providing scientific insights for industries and departments involved in coal transportation.

MODELING OF COAL RAIL-WATER INTERMODAL NETWORK BASED ON SPACE L

China's coal rail-water intermodal network represents an integrated transportation system that facilitates the movement of coal from production regions to consumption regions via railways, inland waterways, and maritime shipping. This system establishes an efficient and secure framework for coal transportation, as illustrated in Figure 1.

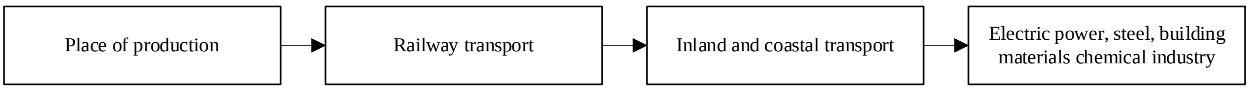


Figure 1. Flow chart of coal rail-water intermodal transportation

Given that the core function of the coal rail-water intermodal network is transportation, this study adopts the Space L model to construct this network. The Space L model captures the position of nodes in the network, the direct connections between nodes, and the network's topological structure. Initially, an undirected and unweighted network $G=(V,E)$ is employed to describe the coal rail-water intermodal infrastructure network, while in the case analysis section, a directed and weighted network $G^{'}=(V^{'},A,W)$ is used to represent the actual operational network of coal rail-water intermodal transportation. Here, $V=\{v_1,v_2,\ldots,v_n\}$ and $V^{'}=\{v_1^{'},v_2^{'},\ldots,v_m^{'}\}$ denote node sets representing coal rail-water intermodal stations. $E=(e_{ij})_{n\times n}$ denotes the infrastructure link between nodes v_i and v_j, where $e_{ij}=1$ if a transportation link exists between nodes v_i and v_j. $A=(a_{ij})_{m\times m}$ denotes the actual transportation link between nodes $v_i^{'}$ and $v_j^{'}$, where $a_{ij}=1$ if coal transportation exists on the link from node $v_i^{'}$ to $v_j^{'}$. $W=(w_{ij})_{m\times m}$ denotes the coal transportation volume on the link from node $v_i^{'}$ to $v_j^{'}$.This study systematically categorizes the main coal transportation channels in China and establishes the topological structure of the coal rail-water intermodal infrastructure network based on the latitude and longitude of nodes, as illustrated in Figure 2.

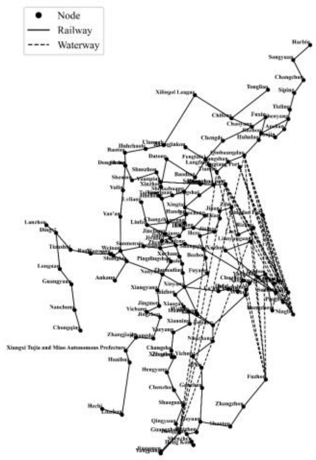


Figure 2. Network topology structure of coal rail-water intermodal transportation infrastructure

A METHOD FOR IDENTIFYING KEY NODES IN COAL RAIL-WATER INTERMODAL NETWORKS INCORPORATING NODE STRENGTH

Development of a Comprehensive Node Importance Evaluation System

In complex networks, nodes play a critical role in transmitting their own "information" and receiving "information" from neighboring nodes within a local scope, while simultaneously impacting the global efficiency of the entire network. Therefore, when assessing the importance of nodes, it is essential to account for both their local and global influences. Considering the unique characteristics of the coal rail-water intermodal network, this study adopts local metrics, including node degree and node strength, as well as the global metric of node efficiency, to comprehensively evaluate node importance. Node degree and node strength are selected as local metrics as they capture the connectivity and interaction patterns of nodes within the coal rail-water intermodal network, highlighting their status and influence within a local context. Node efficiency is chosen as a global metric because it integrates multiple factors, such as node position, connectivity, and network structure. The comprehensive node importance evaluation model developed in this study is illustrated in Figure 3.

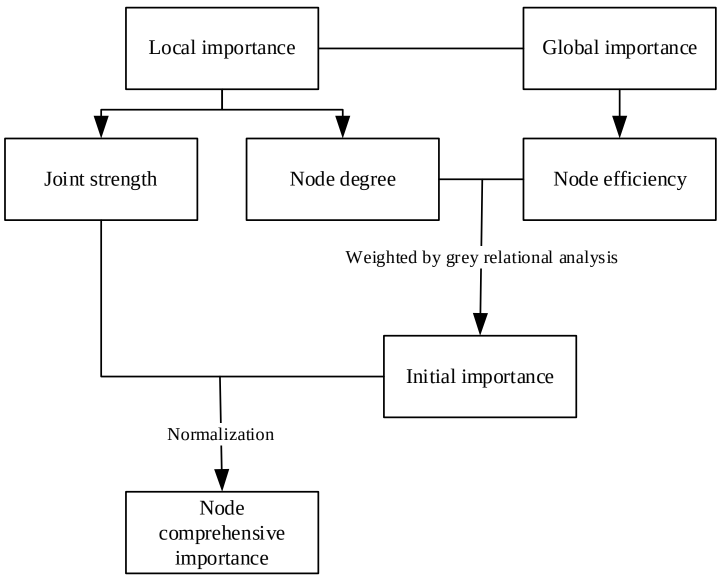


Figure 3. GRAIE-NS method node comprehensive importance evaluation model

Given that the node degree k_i and node efficiency I_i contribute differently to node importance, this study employs the grey relational analysis method to assign weights to these two factors in calculating the initial node importance N_i . To obtain a more comprehensive evaluation of node importance, the node strength S_i is also integrated into the calculation model. The inclusion of node strength broadens the evaluation to include not only the topological structure of the network but also the actual contribution of nodes within the entire coal rail-water intermodal network. By integrating the three metrics—node degree, node efficiency, and node strength—this study enables more accurate identification of key nodes and a better assessment of their critical roles in the network.

$$N_i = \alpha k_i + \beta I_i \tag{1}$$

$$D_i = \eta N_i' + \gamma S_i' \tag{2}$$

$$\eta + \gamma = 1 \tag{3}$$

In the equation: D_i represents the comprehensive importance of the node, α and β denote the weights assigned to node degree and node efficiency, respectively, N_i' and S_i' represent the normalized forms of N_i and S_i ,and η and γ are the weights assigned to N_i' and S_i'.

The definitions and formulas of the metrics utilized in the node importance evaluation framework are presented below.

Node degree

The node degree k_i is defined as the number of edges directly connected to the node. In directed networks, it includes in-degree k_i^{in} and out-degree k_i^{out}. In this context, it represents the railway lines or shipping routes passing through each node. The calculation formula is:

$$k_i = k_i^{in} + k_i^{out} = \sum_{j=1, j\neq i}^{m} a_{ji} + \sum_{j=1, j\neq i}^{m} a_{ij} \tag{4}$$

Node strength

The node strength S_i is defined as the sum of the weights of the edges connected to the node. In directed networks, it includes in-strength S_i^{in} and out-strength S_i^{out}. In this context, it represents the volume of coal transportation flowing through each node. The calculation formula is:

$$S_i = S_i^{in} + S_i^{out} = \sum_{j=1, j\neq i}^{m} w_{ji} + \sum_{j=1, j\neq i}^{m} w_{ij} \tag{5}$$

Node efficiency

The node efficiency I_i is defined as the average reciprocal of the shortest path lengths between the node and all other nodes in the network. It reflects the ease with which the node can transmit information to other nodes in the network. A higher node efficiency value signifies greater centrality and transmission efficiency, suggesting higher global importance of the node in the network[20]. The calculation formula is:

$$I_i = \frac{1}{m-1} \sum_{j=1, j\neq i}^{m} \frac{1}{d_{ij}} \tag{6}$$

In the formula: d_{ij} denotes the shortest path length between nodes v_i' and v_j' (without considering edge weights).

Network Node Strength Calculation Method Based on Coal Turnover Volume

In the network, coal turnover volume captures the scale and efficiency of coal transportation activities, directly influencing the stability and security of national energy supply. Specifically, it is defined as the total of the products of the quantity of goods transported by transportation vehicles and their transportation distance within a specific time period. To maintain national energy security, reduce transportation costs, and mitigate energy price pressures, this study aims to minimize coal turnover volume as the optimization objective, seeking to achieve overall improvement in transportation efficiency while satisfying coal supply-demand requirements and ensuring transportation capacity constraints. Without loss of generality, this study adopts the following assumptions:

Assumption 1: The study does not account for regional differences in coal type demand, assuming uniform requirements for coal types and quality across all regions.

Assumption 2: For dedicated coal transportation lines (where demand locations are not directly served by railway or inland waterway lines, and dedicated transportation lines are established to the nearest cities), it is assumed that their transportation capacity is unlimited.

The sets, parameters, variables, and their descriptions used in the solution process are provided in Table 1.

Table 1. Definitions of sets, parameters and variables

Symbol	Descriptions
Z	Total coal turnover
x_{ij}	Volume of coal transport between node v_i and node v_j
d_{ij}	Distance between node v_i and node v_j
O	Collection of coal production areas
D	Collection of coal demand areas
P_o	Coal output at production area o
M_d	Coal demand at demand area d
N	Complete node set
N_inter	Intermediate node set
N_o	Combined set of production areas and intermediate nodes
N_d	Combined set of intermediate nodes and demand areas
C_{ij}	Capacity of the railway line between nodes v_i and v_j
N_port	Port node set
N_train	Railway station set
C_port_j	Capacity of port j

$$\min z = \sum_{i \in N} \sum_{j \in N} d_{ij} x_{ij} \tag{7}$$

$$\sum_{j \in N, j \neq o} x_{oj} \leq P_o, o \in O \tag{8}$$

$$\sum_{i \in N, i \neq d} x_{id} = M_d, d \in D \tag{9}$$

$$x_{ij} \leq C_{ij}, i \in N\_train, j \in N\_train \tag{10}$$

$$\sum_{i \in N, i \neq j} x_{ij} \leq C\_port_j, j \in N\_port \tag{11}$$

$$\sum_{i \in N\_o} x_{ij} = \sum_{h \in N\_d} x_{jh}, j \in N\_inter \tag{12}$$

$$x_{ij} = 0, i = j 且 i, j \in N \tag{13}$$

$$x_{ij} \times x_{ji} = 0, i \in N, j \in N \tag{14}$$

$$x_{ij} \geq 0 \tag{15}$$

To minimize coal transportation costs, this paper formulates an optimization problem where the objective function, given in Equation (7), aims to minimize the total coal turnover. Equation (8) constrains the coal output from production regions to not exceed their production capacity. Equation (9) ensures that the coal input to consumption regions matches their demand. Equation (10) enforces that the actual transported volume along railway coal transport routes does not exceed their transportation capacity. Equation (11) restricts the coal volume handled at transportation ports to remain within their load capacity. Equation (12) ensures that the total coal input at intermediate nodes equals the total output, maintaining flow conservation. Equation (13) prevents self-loops by ensuring that coal-producing regions do not transport coal to themselves after satisfying their own demand. Equation (14) guarantees that coal transportation occurs in only one direction between any two nodes. Equation (15) imposes a non-negativity constraint on coal transportation volumes. By solving this optimization problem, the actual load of the transportation routes can be determined, which in turn facilitates the calculation of node strength.

Based on the computed node degree, node efficiency, and node strength, the comprehensive importance of each node can be further determined. The specific procedure of the GRAIE-NS method is illustrated in Figure 4.

CASE STUDY

Analysis of the Comprehensive Importance of Nodes in the Coal Rail-Water Intermodal Transportation Network

This study defines coal consumption as the total usage of coal and its derivatives to provide a comprehensive representation of coal resource utilization. To ensure data accuracy and reliability, the data utilized in this study are sourced from official statistical yearbooks, local government reports, and the official websites of provincial statistical bureaus. Given the volatility of coal-related data and the challenges associated with data acquisition, this study employs imputation techniques to estimate missing values, thereby maintaining data continuity and ensuring the completeness of the analysis.

Due to policy regulations and geological constraints, coal production in China is increasingly centralized in the three provinces of Shanxi, Shaanxi, and Inner Mongolia. Over the past decade, the proportion of China's total coal output contributed by these three provinces has steadily increased, rising from 58% in 2010 to 71% in 2023. Moreover, statistical data indicate that the power and steel industries collectively account for over 70% of the country's total coal consumption.

Building on this, this study selects the leading coal-producing cities in Shanxi, Shaanxi, and Inner Mongolia as coal production nodes, while the cities hosting the most coal-intensive power plants and steel mills are designated as coal demand nodes. The annual coal production or consumption of a city serves as the respective production or demand indicator. The transport routes linking production and demand nodes, along with the major cities, railway hubs, and ports they traverse, function as intermediate nodes. This classification results in three node types: production nodes, intermediate nodes, and demand nodes. By integrating the previously described node strength computation method, the infrastructure network for coal transportation in this case study is modeled as a directed weighted rail-water intermodal transportation network, as depicted in Figure 4. The edge weights, expressed as node strength, are detailed in Table 2.

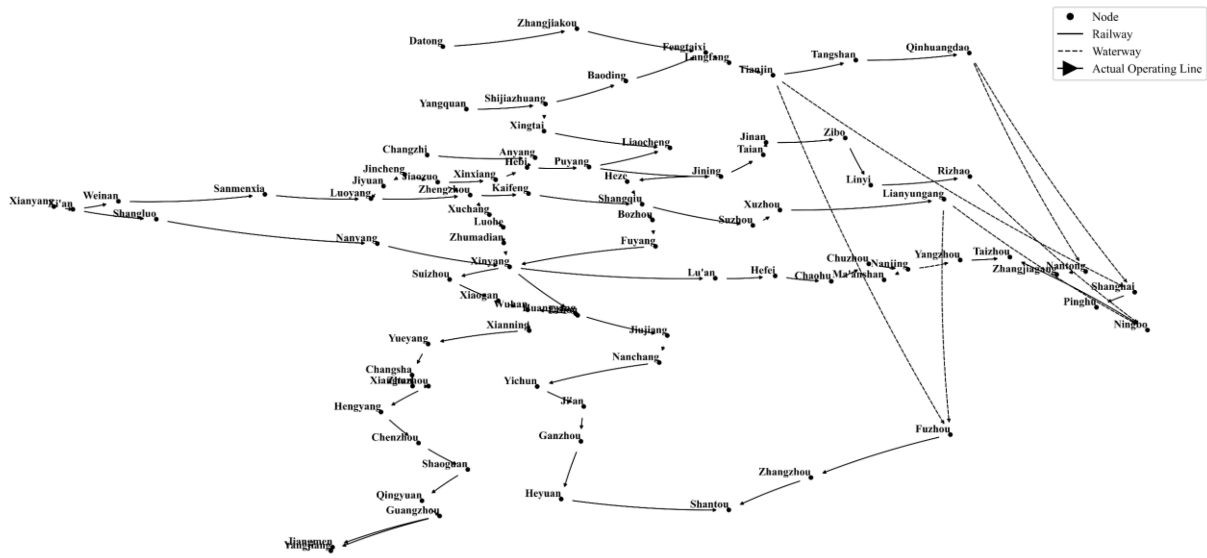


Figure 4. Topological structure of the actual coal rail-water intermodal transportation network

As described earlier in this study, the key node identification framework is first applied to compute the node degree and node efficiency of China's actual coal rail-water intermodal transportation network. Next, the grey relational analysis method is utilized to assign relative weights to node degree and node efficiency in assessing node importance, with determined values of 0.493 and 0.507, respectively. Based on these weights, the initial node importance is calculated. Furthermore, an optimization model is employed to derive the node strength of each node. By combining node strength with initial node importance, the comprehensive importance of each node in the actual operational network is ultimately determined, as presented in Table 2.

When the network topology weight $\eta = 0.2$, the primary management objective of the coal rail-water intermodal transportation network is to ensure the smooth transportation of coal. This scenario is particularly relevant during periods of high national coal demand, such as winter (heating season), where identifying key nodes and formulating corresponding maintenance and emergency response strategies are essential for maintaining the stable operation of the coal transportation network in critical regions.

When the network topology weight $\eta = 0.5$, this weighting is primarily used for the routine identification of key nodes in the coal intermodal transportation network, ensuring its efficient and secure operation under normal demand conditions.

When the network topology weight $\eta = 0.8$, the management focus shifts to emphasizing the overall effectiveness and accessibility of the intermodal transportation network. This scenario applies to challenging operational conditions, such as extreme weather events (e.g., heavy rain, snowstorms) or disruptions (e.g., power supply shortages or localized blackouts), where key node identification is crucial for ensuring the stability and connectivity of the intermodal network.

Table 2. Comprehensive importance of nodes

Node name	Node degree	Node efficiency	Node strength	Initial node importance	Comprehensive node importance $\eta = 0.2, \gamma = 0.8$	Comprehensive node importance $\eta = 0.5, \gamma = 0.5$	Comprehensive node importance $\eta = 0.8, \gamma = 0.2$
Zhangjiakou	2	0.043	23331	1.008	0.700	0.514	0.328
Datong	1	0.042	11665	0.514	0.330	0.209	0.089
Fengtaixi	3	0.045	26094	1.502	0.818	0.661	0.505
Langfang	2	0.049	24357	1.011	0.729	0.533	0.336
Tianjin	4	0.064	19330	2.004	0.665	0.641	0.616
Tangshan	2	0.031	3796.8	1.002	0.145	0.166	0.188
Baoding	2	0.043	4500	1.008	0.165	0.180	0.195
Yangquan	1	0.052	4250	0.519	0.120	0.079	0.038
Changzhi	1	0.081	11897.2	0.534	0.338	0.217	0.097
Shijiazhuang	3	0.060	8500	1.509	0.319	0.350	0.382
……							
Anyang	2	0.086	23794.4	1.029	0.715	0.527	0.338
Hebi	3	0.093	28294.4	1.526	0.882	0.705	0.528
Puyang	3	0.106	28294.4	1.533	0.883	0.706	0.530

Calculations reveal that under three different indicator weightings, 10 out of the top 15 key nodes remain consistent. These key nodes include Puyang, Hebi, Fengtai West, Tianjin, Jining, Xinyang, Zhengzhou, Shangqiu, Jiaozuo, and Shijiazhuang. Most of these nodes function as critical railway hubs or ports, playing a pivotal role in coal logistics, including transportation, handling, and transshipment. Their strategic geographical locations establish them as critical transit points that interconnect different regions, thereby ensuring the seamless and stable operation of the coal transportation network. By linking coal-producing regions, consumption centers, and major ports, these nodes collectively form the backbone of the coal supply chain.

Given their significance, these nodes should be prioritized in network management, with a strong emphasis on enhancing operational capacity, formulating emergency response strategies, and implementing robust maintenance measures. These efforts are essential for sustaining the stability and efficiency of the entire coal intermodal transportation network.

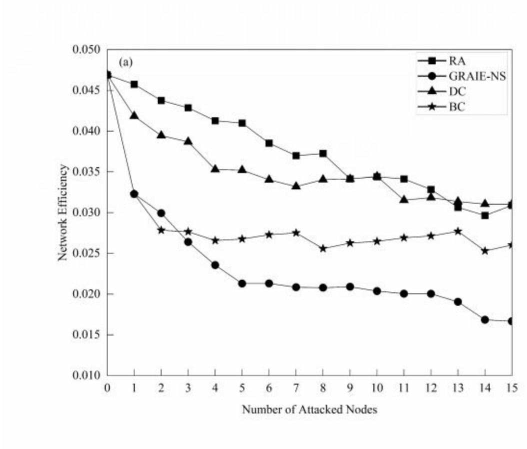
Invulnerability Analysis and Algorithm Comparison

Invulnerability, in a general sense, refers to a network's ability to sustain its functionality despite node or edge failures. Wang et al.[21]define the invulnerability of railway networks as the ability to sustain or rapidly restore an operational state when infrastructure damage or service disruptions occur due to internal or external factors.

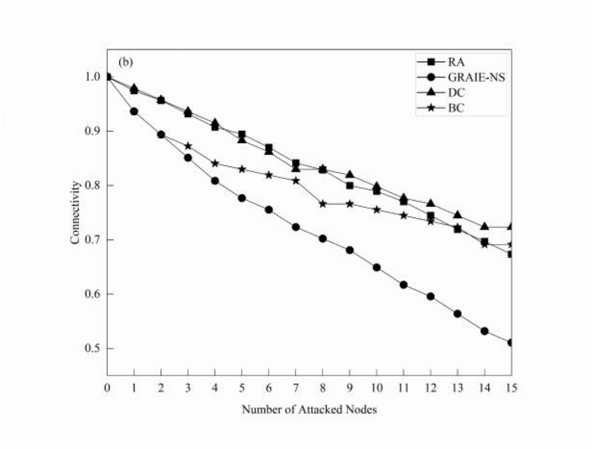
Considering the characteristics of coal transportation networks, this study defines the invulnerability of the coal rail-water intermodal transportation network as its ability to maintain connectivity and continue fulfilling coal transportation demands under disruptions. To evaluate invulnerability, this study utilizes network efficiency and connectivity as the primary measurement indicators[20].

To conduct a comprehensive evaluation of network invulnerability and validate the effectiveness of the GRAIE-NS method, this study compares four attack strategies: (1) random attacks, (2) attacks targeting nodes ranked by the comprehensive importance scores D_i computed using the GRAIE-NS method, (3) attacks targeting nodes based on degree centrality, and (4) attacks targeting nodes based on betweenness centrality. The impact of these attack strategies on network efficiency and connectivity is systematically examined.

Under the random attack strategy, 15 nodes are randomly selected for removal from the network. Multiple simulation trials are performed, and the average result is used as the final measurement outcome. The variations in network performance under different algorithms are illustrated in Figure 5.



(a) Network efficiency variation



(b) Connectivity variation

Figure 5. Network performance variations across different algorithms

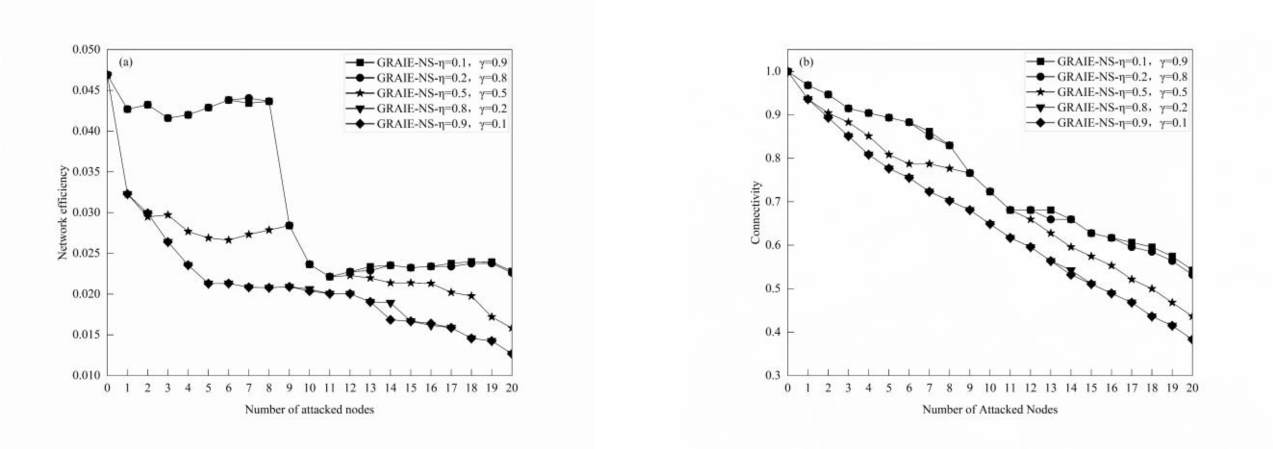
As illustrated in Figure 5, the initial network efficiency is 0.047, and the connectivity is 1. When the network experiences attacks, both network efficiency and connectivity exhibit a decreasing trend. This decline occurs because attacks lead to the removal of critical nodes or edges, thereby weakening network connectivity. The impact is particularly significant when attacks target nodes that occupy structurally important positions in the network topology.

Among the evaluated attack strategies, the approach based on comprehensive importance scores derived from the GRAIE-NS method has a more pronounced effect on both network efficiency and connectivity than conventional strategies, including random attacks, degree-based attacks, and betweenness-based attacks. This further demonstrates the effectiveness of the GRAIE-NS method.

Sensitivity Analysis of Different Indicator Weights

To further investigate how variations in initial node importance and node strength under different weighting schemes influence network invulnerability following key node failures, this study evaluates five different weight combinations: $\eta = 0.1$, $\gamma = 0.9$, $\eta = 0.2$, $\gamma = 0.8$, $\eta = 0.5$, $\gamma = 0.5$, $\eta = 0.8$, $\gamma = 0.2$, and $\eta = 0.9$, $\gamma = 0.1$. For each case, key nodes are identified sequentially and removed from the network, and the variations in network efficiency and connectivity before and after node failures are analyzed. The detailed results are illustrated in Figure 6.

As the weight η assigned to initial node importance increases, the failure of identified key nodes exerts a more pronounced detrimental effect on network performance.



(a) Network efficiency variation

(b) Connectivity variation

Figure 6. Network performance variations under different indicator weights

CONCLUSION

This study constructs a complex network model of coal rail-water intermodal transportation and introduces a novel key node identification method that integrates node strength, aiming to enhance national energy security. The main conclusions are as follows:

(1) Compared with traditional approaches, the key node identification method based on the GRAIE-NS framework has a greater impact on network performance following the failure of identified key nodes, offering a more comprehensive representation of node roles and influence within the network.

(2) Sensitivity analysis was performed by adjusting the weight distribution between initial importance and node strength in the GRAIE-NS method. The results indicate that when a higher weight is assigned to initial importance, the failure of identified key nodes leads to a more pronounced degradation in both network efficiency and connectivity.

This study has made significant progress in exploring the key nodes and invulnerability of the coal rail-water intermodal transportation network, but there are certain limitations, which also point to future research directions:

(1) The coal production and demand locations selected in this study are primarily based on cities with large coal-consuming power plants and steel mills, which facilitates focused analysis. However, this selection does not comprehensively cover all important coal production and demand nodes. Future research should expand the scope to more fully reflect the diversity of the coal transportation network.

(2) This study analyzes the major transportation corridors that dominate coal transportation in China, but the actual transportation network includes many other potential routes. These routes may play a key role under specific conditions. Therefore, future research should consider these additional transportation corridors to enhance the applicability of the model.

Coal, as an important energy resource in China, is unevenly distributed geographically, making coal transportation particularly crucial. Coal rail-water intermodal transportation is an indispensable part of coal transportation. Therefore, scientifically and reasonably identifying key nodes and optimizing the invulnerability of the transportation network is an essential task in this research.

REFERENCES

[1] Hu, W. (2021). Innovative Mode of Rail Water Combined Transportation Based on Smooth “Dual Circulation”. China Transportation Review, 43(11), 97-102.

[2] Li, D. and Zhao, P. (2015). The Analysis of Coal Water-rail Combined Transportation Development Trend in China. Logistics Engineering and Management, 37(04), 3-4.

[3] Sun, Z., Sun, Y., Chang, X., Wang, F., Wang, Q., et al. (2023). Finding critical nodes in a complex network from information diffusion and Matthew effect aggregation. Expert Systems with Applications, 233.

[4] Yin, R., Li, L., Wang, Y., Lang, C., Hao, Z., et al. (2024). Identifying critical nodes in complex networks based on distance Laplacian energy. Chaos, Solitons and Fractals, 180.

[5] Yu, E., Wang, Y., Fu, Y., Chen, D., Xie, M. (2020). Identifying critical nodes in complex networks via graph convolutional networks. Knowledge-Based Systems, 198.

[6] Yang, Y., Wang, X., Chen, Y., Hu, M. (2020). Identifying Key Nodes in Complex Networks Based on Global Structure. IEEE Access, 8, 32904-32913.

[7] Xiang, N., Wang, Q., You, M. (2023). Estimation and update of betweenness centrality with progressive algorithm and shortest paths approximation. Scientific Reports, 13(1), 17110.

[8] Savita, Ankita, V. (2022). Eigen Vector Centrality (EVC) Routing for Delay Tolerant Networks: A Time Associated Matrix-Based Approach. Wireless Personal Communications, 128(2), 1217-1233.

[9] An, Z., Hu, X., Jiang, R., Jiang, Y. (2024). A novel method for identifying key nodes in multi-layer networks based on dynamic influence range and community importance. Knowledge-Based Systems, 305.

[10] Li, P., Wang, S., Chen, G., Bao, C., Yan, G. (2022). Identifying Key Nodes in Complex Networks Based on Local Structural Entropy and Clustering Coefficient. Mathematical Problems in Engineering, 2022.

[11] Tian, B., Hu, X. (2024). Identification of Key Nodes between Overlapping Communities by Using Nonnegative Matrix Factorization: SSRN.

[12] Zhong, L., Gao, X., Zhao, L., Zhang, L., Chen, P., et al. (2023). A hybrid influence method based on information entropy to identify the key nodes. Frontiers in Physics, 11.

[13] Zhang, H., Zhang, S., Xie, X., Zhang, T., Yu, G. (2023). Identification of Key Nodes in Complex Networks Based on Network Representation Learning. IEEE Access, 11, 128175-128186.

[14] Ren, T., Sun, S., Xu, Y., Marko Dimirovski, G. (2024). Key nodes mining for complex networks based on local gravity model. Journal of Control and Decision, 11(3), 409-416.

[15] Li, X., Xu, M., Ai, Q., Cao, W. (2025). Parallel Encoding Method for Critical Node Identification in Transportation Networks. IAENG International Journal of Computer Science, 52(2), 484-490.

[16] Zhao, N., Wang, H., Wen, J., Li, J., Jing, M., et al. (2023). Identifying critical nodes in complex networks based on neighborhood information. New Journal of Physics, 25(8).

[17] Qu, J., Shi, X., Li, M., Cai, Y., Yu, X., et al. (2025). Identifying critical nodes in multiplex complex networks by using memetic algorithms. Physics Letters, Section A: General, Atomic and Solid State Physics, 529.

[18] Fu, L., Ma, X., Dou, Z., Bai, Y., Zhao, X. (2024). Key Node Identification Method Based on Multilayer Neighbor Node Gravity and Information Entropy. Entropy, 26(12), 1041.

[19] Lei, M., Liu, L., Ramirez-Arellano, A. (2024). Weighted information index mining of key nodes through the perspective of evidential distance. Journal of Computational Science, 78.

[20] Wang, T., Zhang, Y., Zhou, M., Lu, W., Li, S. (2022). Identification of Key Nodes of Urban Rail Transit Integrating Network Topology Characteristics and Passenger Flow. Journal of Transportation Systems Engineering and Information Technology, 22(06), 201-211.

[21] Wang, W., Liu, J., Li, H., Jiang, X. (2010). Survivability Analysis of Railway Network. Journal of the China Railway Society, 32(04), 18-22.