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Outsourcing and Value Chain Climbing: Vertical Coopetition in Buyer-supplier Supply Chains under Demand Uncertainty When the Supplier is Powerful

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Abstract:
We study the supplier's value chain climbing behavior in a two-level supply chain consisting of a buyer and a supplier, where the buyer outsources its production to the supplier in the first stage, and the supplier may enter the market and compete with the buyer in the second stage. We model the problem as a two-stage game, assuming the supplier is relatively powerful and sets the outsourcing price. We show how the buyer decides on the outsourcing strategy and/or in-house production quantities, and obtain three equilibria, depending on the attitudes of the buyer to the supplier's value chain climbing behavior: (1) complete outsourcing in Stage 1; (2) complete in-house production in Stage 1; and (3) partial outsourcing in Stage 1. We identify a few properties of each equilibrium and examine if outsourcing reduces production costs under various conditions. Our computational study analyzes the optimal outsourcing strategies, expected profits, and the impact of demand uncertainty. The computational results indicate that a buyer may switch from outsourcing to in-house production if product prices are high or production costs are low. Additionally, the demand distribution affects both the speed and direction of this shift.

Keywords: Value Chain Climbing; Outsourcing; Two-Stage Game; Vertical Coopetition; Demand Uncertainty

INTRODUCTION

Value chain climbing is crucial for suppliers focused on production. Firms often outsource manufacturing to suppliers like OEMs, enabling technology giants like Apple and Intel to prioritize core activities such as R&D and marketing, thus boosting competitiveness. Outsourcing also offers financial flexibility and capabilities not internally available. Conversely, suppliers can develop capabilities to compete with buyers by interacting with them, holding core technology or IP rights [1]. This phenomenon, termed value chain climbing, is a significant route for industrial upgrading in developing countries [2]. Samsung's entry into the smartphone market exemplifies value chain climbing. When Apple launched the iPhone 4.0, it selected Samsung as a key component supplier due to Samsung's technological prowess and low development costs. This partnership allowed Apple to reduce production costs while ensuring iPhone quality, and it enabled Samsung to gain insights into smartphone design and manufacturing. Leveraging its manufacturing strengths, Samsung swiftly introduced the Galaxy series in 2011, becoming a global smartphone competitor. This move significantly affected Apple's market share and profits, leading to a patent infringement lawsuit against Samsung in April, spanning Europe, Asia, and Australia.

While Samsung and other suppliers like Galanz and HTC have profited from competing with their customers through value chain climbing, many suppliers stick to supply-only roles to maintain buyer relationships. Suppliers avoid value chain climbing mainly due to fear of buyer retaliation, such as being replaced or facing entry barriers [3], and dependence on buyers, which limits their technological advancement and market competitiveness [4,5]. Although much research focuses on supply chain strategies of suppliers and buyers, our study uniquely examines operational factors influencing these strategies' success, an area often overlooked in supply chain strategic management. For example, we explore how demand uncertainty significantly impacts firm revenue and how suppliers and buyers can use this insight to enhance their business strategies. Our findings suggest that suppliers may not compete with buyers if demand is stable but may enter the market if it rapidly grows and presents promising opportunities.

Some suppliers have gained power over outsourcing buyers in various industries due to their large capacity and technological capability, challenging the traditional view in neoclassical economics where the buyer is more powerful and the supplier is weaker [6]. For example, an expert at a well-known electronics company noted that their contracts with OEM customers offer little room for negotiating per-unit prices, reflecting the new reality of powerful contract manufacturers [7]. In this study, we assume the supplier is the first mover in the supply chain. We explore the vertical coopetition game under demand uncertainty when the supplier leads the supply chain and the game between the buyer and the supplier is quantity-based. Specifically, we address the question: How does the buyer determine outsourcing/in-house quantities in a two-stage game when the supplier sets the outsourcing price first?

The remainder of the paper is organized as follows. Following a brief literature review in Section 2, we describe the setting of the problem and construct a game model in Section 3. We then present our analysis of the model in Section 4. Using computational study, we demonstrate various findings in Section 5. We conclude the study with a discussion of the implications and future research in Section 6. The proofs of the Theorems and Lemmas are relegated to the Appendix.

LITERATURE REVIEW

Outsourcing and Value Chain Climbing in the Supply Chain

The value chain climbing is influenced by several factors [8,9]: 1. The rising disposable income in developing countries has created significant domestic markets for suppliers to start their businesses before expanding globally [10,11]. 2. Emerging competitors can sell products to developing countries before entering developed markets to gain experience and higher profits [12]. 3. Recent advancements like e-commerce, 5G, and blockchain have reduced entry barriers for suppliers, allowing them to move up the value chain. 4. Established companies rely on suppliers for production advantages, and buyers still cooperate with them due to the lack of substitutes [13].

Multiple suppliers have attempted to launch their own branded products using learning technology, but only a few have succeeded [14,15]. Major buyers employ stringent outsourcing contracts to maintain suppliers in a subordinate position and away from customer-facing tasks [2]. These factors indicate that outsourcing will continue to grow, benefiting companies with a strategic approach [16]. Buckley and Verbeke [17] indicate suppliers with higher transactional dependence on buyers are less likely to invest in technological upgrading. According to [5], the decision to refrain from upgrades is influenced by firm-level technological resources, industrylevel technological intensity, and regional technological protection. Previous studies [2,18,19] have shown that suppliers climbing value chains can lead to vertical competition in the supply chain, advising caution for Western companies dealing with suppliers in developing countries. Wan and Wu [8] developed a model to examine how this movement affects buyer-supplier relationships. They discuss three outsourcing strategies: accommodation, squeeze, and dump, and how the buyer's optimal strategy and vertical relationships may change in response to the supplier's gradual capability development. However, their analysis ignores operational factors, such as demand and inventory, and the uncertain operational environment may lead to strategy failure.

Outsourcing Contract in Supply Chain Management

The literature on outsourcing and supply chain contracting often explores how different factors affect outsourcing decisions. The most common factors include learning curve of the production cost [20], contract type [21,22] and product substitution [23,24]. Other factors, such as scale economies [25], capacity pooling [26], and timing of market entry [27,28] are all explored. A substantial body of literature on outsourcing contracts examines the buyer's decision to make or buy [29]. Most relevant literature focuses on quantity-based outsourcing contracts and the decision to outsource production supplier(s) in various scenarios [30,31]. Despite production cost, bargaining power [32], and product quality [33,34] also affect outsourcing decisions. Scholars have suggested considering outsourcing and strategic decisions together. For example, a branded firm can sell its product to the end market and act as a supplier to another branded firm [35]. Caldieraro [23] examines the joint decision of market entry and outsourcing for a firm in a horizontally and vertically differentiated market. Liu et al. [36] explore outsourcing strategies with patent licensing in a supply chain. Ghamat et al. [37] discuss how limited supplier or CM capacity leads to vertical and horizontal competition.

Competition setting in outsourcing relationships

The existing literature on outsourcing contracts focuses on wholesale pricing based on quantity-based competition [28,38]. The buyer is typically considered the first mover [23,39], but in some cases, the supplier may have more influence and set the wholesale price first [40,41]. Some scholars studied the outsourcing quantity decisions in Cournot competition [42]. Wang et al. [43] compared Stackelberg and Cournot competitions and found more precise outsourcing strategies. Additionally, scholars explored outsourcing contracts through the bargaining process [44] or negotiation process [45]. In addition to quantity contracts, many scholars focus on price competition in outsourcing strategies. Niu et al. [46] examine price competition between an OEM and its ODM in various scenarios. Chen et al. [47] discuss how the attitudes of OEM and ODM toward risk influence their outsourcing behaviors in different price competition games. Shi [48] explores the CM's encroachment strategy and quality decisions based on price competition.

The uncertainties in outsourcing development, particularly in demand, supply, capacity [49,50], and asymmetric outsourcing information [7] are receiving more attention. Especially, Yan et al. [42] establish a multistage Bayesian game model to analyze outsourcing decisions between an OEM and a CM. Several scholars study different aspects of outsourcing strategies, including the optimal timing of capacity investment or launching product [42,51,52], buyer's outsourcing strategies considering supplier's competition and uncertain cost [49,53].

Our research focuses on outsourcing decisions under demand uncertainty and the coopetition between a buyer and a supplier within value chain integration. We examine quantity-based outsourcing contracts with the supplier leading the decision process, consisting of two stages. In Stage 1, the supplier learns from the buyer by outsourcing if the outsourcing relationship exists. In Stage 2, the supplier decides whether to enter the market with their own branded product. Stochastic demands are involved in both stages.

MODEL SETTING AND FORMULATION

In this paper, a supply chain consists of a buyer (she, b) and a supplier (he, s) is considered. First, the buyer exclusively sells the product at p. The buyer can choose to produce the product herself or outsource to the supplier. If outsourced, the supplier may gain access to the buyer's knowledge and compete with her, known as value chain climbing. We divide the timeline into two stages: Stage 1 is the learning stage: the supplier cannot provide products to the market, as he needs to improve the corresponding capabilities from value chain climbing. Stage 2 is the competition stage: the supplier can provide products to the market and compete with the buyer if the outsourcing relationship exists in Stage 1. For convenience, Table 1 lists the notations and decision variables.

We conceptualize the supplier's and buyer's sequential decisions in Figure 1. First, the supplier determines the outsourcing price w. Then, the buyer sets outsourcing quantity $\mathrm{q_{out1}}$and in-house production quantity $\mathrm{q_{in1}}$ facing stochastic demand $\mathrm{D_1}$ in Stage 1. The last step is that the buyer determines her outsourcing quantity $\mathrm{q_{out2}}$, and the supplier simultaneously sets his production quantity $\mathrm{q_s}$ facing stochastic demand $\mathrm{D_2}$ in Stage 2. In the next part, we will discuss the situation when $\mathrm{w < c_s}$ appears in the outsourcing relationship between Samsung and Apple.

Table 1. Notations and decision variables

Notations	
p	product's price
c_b	the buyer's unit production cost
c_s	the supplier's unit production cost, $0 < c_s < c_b < p < 1$
D_1, D_2	the stochastic demands in Stage 1 and Stage 2, respectively, $D_1 \in (0,1), D_2 \in (0,1)$
$F_1(\cdot), F_2(\cdot)$	the cumulative distribution function (cdf) for D_1 and D_2, respectively
$f_1(\cdot), f_2(\cdot)$	the probability density functions (pdf) for D_1 and D_2, respectively
$\alpha(q_{\text{out }1})$	α is the supplier's market share in Stage 2, which is the an infinitely differentiable function of $q_{\text{out }1}, \alpha(q_{\text{out }1}) \in [0,1), \alpha(0) = 0, \frac{\mathrm{d}\alpha(q_{\text{out }1})}{\mathrm{d}q_{\text{out }1}} > 0, \frac{\mathrm{d}^2\alpha(q_{\text{out }1})}{\mathrm{d}q_{\text{out }1}} < 0$
Decision variables	
w	the outsourcing price, $0 < c_s \le w \le c_b < 1$
$q_{\text{out }1}$	the buyer's outsourcing quantity in Stage 1, $0 \le q_{\text{out }1} \le 1$
$q_{\text{in }1}$	the buyer's in-house production quantity in Stage 1, $0 \le q_{\text{in}1} \le 1$
$q_{\text{out }2}$	the buyer's outsourcing quantity in Stage 2, $0 \le q_{\text{out }2} \le 1$
q_s	the supplier's production quantity in Stage 2, $0 \le q_s \le 1$

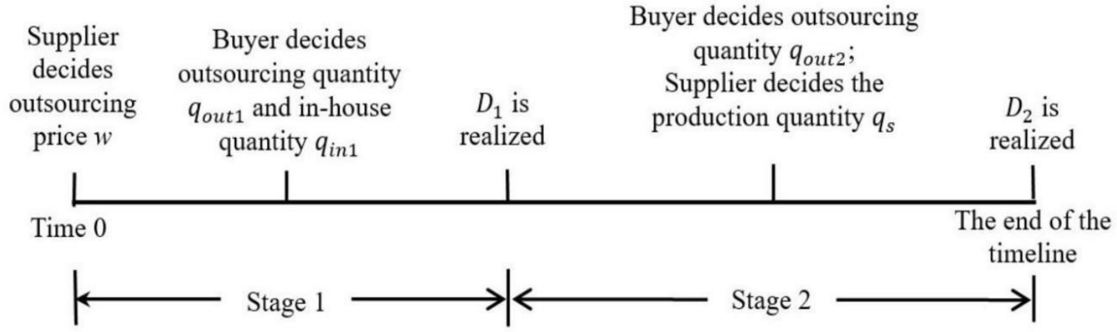


Figure 1. Sequence of events

In Stage 1, we use the newsvendor setting to describe the expected profit of the buyer given:

$$\pi_b^1(w, q_{out1}, q_{in1}) = pE[\min(q_{out1} + q_{in1}, D_1)] - wq_{out1} - c_b q_{in1} \tag{1}$$

The expected profit of the supplier in this stage is:

$$\pi_s^1(w, q_{out1}) = (w - c_s)q_{out1} \tag{2}$$

In Stage 2, α is introduced to describe the supplier's value chain climbing performance. A larger $q_{\text{out }1}$ means the supplier is more likely to obtain technology or market information from the buyer, enhancing its capabilities in Stage 1 and capturing more market share in the next stage $\left(\frac{\mathrm{d}\alpha(q_{\text{out }1})}{\mathrm{d}q_{\text{out }1}} > 0\right)$. Meanwhile, the law of diminishing marginal benefit is also applied: as q_{out1} increases, the rate of increase in market share for the supplier in Stage 2 decreases $\left(\frac{\mathrm{d}^2\alpha(q_{\text{out }1})}{\mathrm{d}q_{out1}^2} < 0\right)$. Assume that if the buyer does not outsource to the supplier in Stage 1, the supplier will not enter the market in Stage 2. Note that the supplier can self-invest in Stage 1 and enter the market in Stage 2, actively becoming the buyer's competitor. Then, we write the buyer's expected profit in Stage 2 as:

$$\pi_b^2(w, q_{\text{out }1}, q_{\text{out }2}) = pE\left[\min\left(q_{\text{out }2}, \left(1 - \alpha(q_{\text{out }1})\right)D_2\right)\right] - wq_{\text{out }2} \tag{3}$$

and the supplier's expected profit in Stage 2 as:

$$\pi_s^2(w, q_{\text{outm }1}, q_{\text{out }2}, q_s) = (w - c_s)q_{\text{out }2} + pE[\min(q_s, \alpha(q_{\text{out }1})D_2)] - c_s q_s \tag{4}$$

The total expected profit of the buyer is

$$\Pi_b(w, q_{out1}, q_{\text{in }1}, q_{out2}) = \pi_b^1(w, q_{out1}, q_{in1}) + \pi_b^2(w, q_{out1}, q_{out2}) \tag{5}$$

the total expected profit of the supplier is

$$\Pi_s(w, q_{out1}, q_{out2}, q_s) = \pi_s^1(w, q_{out1}) + \pi_s^2(w, q_{out1}, q_{out2}, q_s) \tag{6}$$

Observing Equations (3) to (4), we found that $q_{\text{out }2}$ and q_s can be obtained by solving two separate newsvendor models, and Lemma 1 shows the best response of the buyer and supplier in Stage 2 given the decision variables (w and $q_{\text{out }1}$) in Stage 1.

Lemma 1. Given w and q_{out1}, the best responses of the buyer and the supplier in Stage 2 are:

$$\begin{cases} q_{out2}^*(w, q_{\text{out }1}) = \left(1 - \alpha(q_{\text{out }1})\right)F_2^{-1}\left(\frac{p-w}{p}\right) \\ q_s^*(q_{\text{out }1}) = \alpha(q_{\text{out }1})F_2^{-1}\left(\frac{p-c_s}{p}\right) \end{cases} \tag{7}$$

Now, there are three decision variables remaining: q_{out1}, q_{in1} and w. We rewrite the buyer's expected profit given $\left(q_{\text{out }2}^*(w, q_{out1}), q_s^*(q_{\text{out }1})\right)$ to be

$$\begin{aligned} \Pi_b(w, q_{out1}, q_{\text{in1 }1}) &:= \pi_b^1(w, q_{out1}, q_{\text{in1}}) + \pi_b^2\left(w, q_{\text{out }1}, q_{out2}^*(w, q_{\text{out }1})\right) \\ &= pE[\min(q_{out1} + q_{\text{in }1}, D_1)] - wq_{out1} - c_b q_{\text{in }1} \end{aligned} \tag{8}$$

Similarly,

$$\begin{aligned} \Pi_s(w, q_{\text{out }1}) &:= \pi_s^1(w, q_{\text{out }1}) + \pi_s^2\left(w, q_{\text{out }1}, q_{out2}^*(w, q_{\text{out }1}), q_s^*(q_{\text{out }1})\right) \\ &+\alpha(q_{\text{out }1})\left\{pE\left[\min\left(F_2^{-1}\left(\frac{p-c_s}{p}\right), D_2\right)\right] - c_s F_2^{-1}\left(\frac{p-c_s}{p}\right)\right\} \end{aligned} \tag{9}$$

Then, the buyer's decision problem in Stage 1 can be represented as:

$$\max_{q_{out1}, q_{\text{in1}} \geq 0} \Pi_b(w, q_{\text{out }1}, q_{\text{in }1}) \tag{10}$$

Define $q_{out1}^*(w), q_{in1}^*(w)$ as the optimal solutions to Problem (10). Plugging $q_{out1}^*(w)$ into $\Pi_s(w, q_{out1})$, the supplier's optimization problem is

$$\max_{c_s \leq w \leq c_b} \Pi_s(w, q_{out1}^*(w)) \tag{11}$$

Denote w^* as the optimal solution to the Problem (11). Thus, an equilibrium of the whole game is

$$\left(w^*, q_{out1}^*(w^*), q_{in1}^*(w^*), q_{out2}^*\left(w^*, q_{out1}^*(w^*)\right), q_s^*\left(q_{out1}^*(w^*)\right)\right)$$

EQUILIBRIUM ANALYSIS

In this section, we analyze the properties of the equilibrium. Obviously, Given $w, q_{out1}^*(w) + q_{in1}^*(w) > 0$ for $\Pi_b(w, 0, 0) < \Pi_b(w, 0, q_{in1}), \forall q_{in1} \in (0,1)$. Now, we analyze the optimal solution to the Problem (10). Because $\Pi_b(w, q_{\text{out }1}, q_{\text{in }1})$ is continuous in the domain $[c_s, c_b] \times [0,1] \times [0,1]$, we can determine $q_{out1}^*(w) + q_{in1}^*(w) > 0$ by exhaustively searching over all stationary points and boundary points. Let $\pi_{b2}(w):=pE\left[\min\left(F_2^{-1}\left(\frac{p-w}{p}\right),D_2\right)\right]-wF_2^{-1}\left(\frac{p-w}{p}\right)>0$, which is a newsvendor problem's optimal value. Based on these results, **Theorem 2** shows the properties of optimal solution to Problem (10):

Theorem 2. The optimal solution of Problem (10), $(q_{out1}^*(w),q_{in1}^*(w))$, satisfies: (a) $0\le q_{out1}^*(w)<F_1^{-1}\left(\frac{p-w}{p}\right),0\le q_{in1}^*(w)\le F_1^{-1}\left(\frac{p-c_b}{p}\right),q_{out1}^*(w)+q_{in1}^*(w)\le F_1^{-1}\left(\frac{p-c_s}{p}\right)$; (b) If $J_1(q_{\text{out }1},q_{\text{in }1})\ge 0$, where

$$J_1(q_{out1},q_{\text{in }1}):=\inf\left\{-pf_1(q_{out1}+q_{\text{in }1})-\frac{\mathrm{d}^2\alpha(q_{out1})}{\mathrm{d}q_{\text{out }1}^2}\pi_{b2}(c_b)\middle|\ 0<q_{out1}<F_1^{-1}\left(\frac{p-w}{p}\right),\right.$$
$$\left.0\le q_{\text{in }1}<F_1^{-1}\left(\frac{p-c_b}{p}\right),0\le q_{out1}+q_{\text{in }1}\le F_1^{-1}\left(\frac{p-c_s}{p}\right),c_s\le w\le c_b\right\}$$

then $q_{out1}^*(w)=0,q_{in1}^*(w)>0$.

Theorem 2 implies three types of optimal solutions to the Problem (10) reflecting the buyer's trade-off between cost advantage and value chain climbing:

1) $q_{out1}^*(w)=0,q_{in1}^*(w)>0$, the buyer only produces in-house in Stage 1 to maintain the entire market share in Stage 2. It is the boundary solution to avoid value chain climbing;

2) $q_{out1}^*(w)>0$, the buyer outsources in Stage 1 to allow the supplier to become her competitor in Stage 2. There are two sub-cases:
$-q_{out1}^*(w)>0,q_{in1}^*(w)=0$, the buyer only outsources in Stage 1 and ignores the threat of value chain climbing; $-q_{out1}^*(w)>0,q_{in1}^*(w)>0$, the buyer simultaneously produces in-house and outsources in Stage 1. This is the equilibrium balancing the cost advantage and value chain climbing.

Next, we discuss the above cases separately.

Production In-house Only

Given $q_{out1}^*(w)=0,q_{\text{in }1}^*(w)>0$, we first define the failure rates of D_1,D_2 by $r_1(\xi):=f_1(\xi)/(1-F_1(\xi))$ and $r_2(\xi):=f_2(\xi)/(1-F_2(\xi))$, respectively. Note that $F_i(\xi)$ is an increasing failure rate (IFR) distribution if $r_i(\xi)$ is weakly increasing for all ξ such that $F_i(\xi)<1,i\in\{1,2\}$ (Lariviere (2006)). Then, Theorem 3 shows the equilibrium with $q_{out1}^*(w)=0,q_{in1}^*(w)>0$.

Theorem 3. If only producing in-house in Stage 1 is optimal for the buyer, the best responses of the buyer and supplier are $q_{\text{out }1}^*(w^*)=0,q_{\text{in }1}^*(w^*)=F_1^{-1}\left(\frac{p-c_b}{p}\right),q_{out2}^*(w^*,0)=F_2^{-1}\left(\frac{p-w^*}{p}\right)$, and w^* satisfies: (a) $w^*>c_s$; (b) If $F_2(\cdot)$ is an IFR distribution function satisfying

$$r_2(\phi)\le\frac{2p}{c_b-c_s},\forall\phi\in(0,1)$$

then $\Pi_s(w,0)$ is concave in $w\in[c_s,c_b]$, and (1) w^* is the solution to $g(w):=pf_2\left(\frac{p-w}{p}\right)F_2^{-1}\left(\frac{p-w}{p}\right)-w+c_s=0,c_s<w<c_b$ when $g(c_b)<0$, or (2) $w^*=c_b$ when $g(c_b)\ge 0$. (c) If the condition in (b) does not hold, then exhaustively searching over all values of $w\in[c_s,c_b]$ satisfying $g(w)=0$ and boundary point $w=c_b$ will determine w^*.

Theorem 3(a) shows the sufficient conditions under which $\Pi_s(w,0)$ is concave in w, ensuring the uniqueness of w^*. Otherwise, we need to search over all stationary points and boundary points $w=c_b$ to determine w^*, as suggested in Theorem 3(b). The function $g(w)$ is derived from first-order derivation $\mathrm{d}\Pi_s(w,0)/\mathrm{d}w$ with the same sign.

The equilibrium solution shown in Theorem 3 indicates that when the buyer's best strategy is preventing value chain climbing to maintain the whole market share, she may still obtain a price advantage from outsourcing ($c_s<w^*<c_b$). However, the supplier may also set the highest outsourcing price ($w^*=c_b$), illustrating that producing entirely in-house in both stages may also be optimal. This equilibrium explains why some high-tech companies, such as Intel, did not outsource manufacturing despite the availability of many qualified suppliers with lower production costs.

Outsourcing in Stage 1

We now discuss the cases in which the buyer outsources in Stage 1, i.e., $q_{out1}^*(w)>0$. According to the above discussion, there are two sub-cases: (1) $\mathrm{q}_{\text{out1}}^*(\mathrm{w})>0,\mathrm{q}_{\text{in1}}^*(\mathrm{w})=0$, the buyer ignores the value chain climbing of the supplier, and

(2) $q_{out1}^*(w) > 0, q_{in1}^*(w) > 0$, the buyer trades off between the cost advantage of outsourcing and the risk from value chain climbing.

We first consider the buyer's decision in Problem (10). When $q_{in1}^*(w) = 0$, the buyer's objective function in Stage 1 becomes $\Pi_b(w, q_{out1}, 0)$. Generally, $q_{out1}^*(w)$ is determined by searching all stationary points, i.e. when $q_{in1}^*(w) = 0$,

$$q_{out1}^*(w) := \arg\max\left\{\Pi_b(w, q_{out1}, 0) \middle| p[1 - F_1(q_{out1})] - w - \frac{\mathrm{d}\alpha(q_{out})}{\mathrm{d}q_{out}1}\pi_{b2}(w) = 0,\right.$$
$$\left. -pf_1(q_{out1}) - \frac{\mathrm{d}^2\alpha(q_{out1})}{\mathrm{d}q_{out1}^2}\pi_{b2}(w) \le 0, 0 \le q_{out1} \le F_1^{-1}\left(\frac{p-w}{p}\right)\right\}.$$

Lemma 4 illustrates a sufficient condition to ensure the uniqueness of $q_{out1}^*(w)$ given $q_{in1}^*(w) = 0$.

Lemma 4. Given $q_{\text{in }1}^*(w) = 0$, if $J_2(q_{out1}) \le 0$ and $J_3(w, q_{out1}) > 0$, then $\Pi_b(w, q_{\text{out }1}, 0)$ is concave in $q_{\text{out }1} \in \left(0, F_1^{-1}\left(\frac{p-w}{p}\right)\right)$ and $q_{\text{out }1}^*(w)$ is unique, where

$$J_2(q_{out1}) := \sup\left\{-pf_1(q_{\text{out }1}) - \frac{\mathrm{d}^2\alpha(q_{\text{out }1})}{\mathrm{d}q_{\text{out }1}^2}\pi_{b2}(c_s) \middle| \ 0 < q_{\text{out }1} \le F_1^{-1}\left(\frac{p-c_s}{p}\right)\right\}$$
$$J_3(w, q_{\text{out }1}) := \inf\left\{p - w - \frac{\mathrm{d}\alpha(q_{\text{out }1})}{\mathrm{d}q_{\text{out }1}}\pi_{b2}(w) \middle| \ q_{\text{out }1} = 0, c_s \le w \le c_b\right\}$$

Given $\frac{\partial\Pi_b(w,q_{out1},q_{in1})}{q_{\text{in}}} = p[1 - F_1(q_{out1} + q_{in1})] - c_b = 0$, we know that if $q_{in1}^*(w) > 0$, then $(q_{out1}^*(w), q_{in1}^*(w))$ satisfies

$$q_{out1}^*(w) + q_{in1}^*(w) = F_1^{-1}\left(\frac{p-c_b}{p}\right)$$

Thus, we can define

$$q_{\text{out }1}^*(w) := \arg\max\left\{\Pi_b\left(w, q_{\text{out }1}, F_1^{-1}\left(\frac{p-c_b}{p}\right) - q_{\text{out }1}\right) \middle| \ c_b - w - \mathrm{d}\alpha(q_{\text{out }1})/\mathrm{d}q_{\text{out }1}\pi_{b2}(w) = 0,\right.$$
$$\left. -pf_1\left(F_1^{-1}\left(\frac{p-c_b}{p}\right)\right) - \mathrm{d}^2\alpha(q_{\text{out }1})/\mathrm{d}q_{\text{out }1}^2 \le 0, \le q_{\text{out }1} \le F_1^{-1}\left(\frac{p-w}{p}\right)\right\}.$$

Now we discuss the supplier's choice. Given $q_{out1}^*(w) > 0$, the optimal solution to Problem (11), w^*, can be determined by searching all stationary points and the boundary point $w = c_s$. Note that given $q_{out1}^*(w) > 0, w^* \ne c_b$. We can verify this by contradiction: if $w^* = c_b$, then $\Pi_b(c_b, q_{out1}^*(w), q_{in1}) < \Pi_b(c_b, 0, q_{in1}), \forall q_{in1} \in [0,1]$, and thus $q_{out1}^*(w) > 0$ is impossible, indicating that the buyer will not outsource in Stage 1 when considering the loss caused by value chain climbing in Stage 2 if outsourcing has no cost advantage.

Directly analyzing the derivatives of $\Pi_s(w, q_{out1}^*(w))$ is complex. To simplify the analysis, we define $g_1(w, q_{out1})$ by replacing $q_{out1}^*(w)$ with q_{out1} in $\mathrm{d}q_{out1}^*(w))/\mathrm{d}w$:

$$\frac{\mathrm{d}q_{out1}^*(w)}{\mathrm{d}w} = g_1(w, q_{out1})|_{q_{out1}=q_{out1}^*(w)}$$

Then, we define $g_2(w, q_{out1})$ by replacing $q_{out1}^*(w)$ with q_{out1} and substituting $g_1(w, q_{out1})$ with $\mathrm{d}q_{out1}^*(w))/\mathrm{d}w$ in $\mathrm{d}^2q_{out1}^*(w))/\mathrm{d}w^2$:

$$\frac{\mathrm{d}^2q_{out1}^*(w)}{\mathrm{d}w^2} = g_2(w, q_{out1})|_{q_{out1}=q_{out1}^*(w), g_1(w,q_{out1})=\mathrm{d}q_{out1}^*(w)/\mathrm{d}w}$$

Furthermore, we define $G_1(w, q_{out1})$ and $G_2(w, q_{out1})$ by replacing $q_{out1}^*(w)$ with $q_{out1}, g_1(w, q_{out1})$ with $\mathrm{d}q_{out1}^*(w))/\mathrm{d}w$ and $g_2(w, q_{out1})$ with $\mathrm{d}^2q_{out1}^*(w))/\mathrm{d}w^2$ in $\mathrm{d}q_{out1}^*(w))/\mathrm{d}w, \mathrm{d}\Pi_s(w, q_{out1}^*(w))/\mathrm{d}w$ and $\mathrm{d}^2\Pi_s(w, q_{out1}^*(w))/\mathrm{d}w^2$, respectively. That is,

$$\frac{\mathrm{d}\Pi_s(w, q_{out1}^*(w))}{\mathrm{d}w} = G_1(w, q_{out1})|_{q_{out1}=q_{out1}^*(w), g_1(w,q_{out1})=\mathrm{d}q_{out1}^*(w)/\mathrm{d}w},$$
$$\frac{\mathrm{d}^2\Pi_s(w, q_{out1}^*(w))}{\mathrm{d}w^2} = G_2(w, q_{out1})|_{q_{out1}=q_{out1}^*(w), g_1(w,q_{out1})=\mathrm{d}q_{out1}^*(w)/\mathrm{d}w, g_2(w,q_{out1})=\mathrm{d}^2q_{out1}^*(w)/\mathrm{d}w^2}.$$

According to Theorem 2(b), $J_1(q_{out1}, q_{in1}) < 0$ is necessary to the solution $q^*(w) > 0, q_{in1}^*(w) = 0$ (converse-negative proposition), Lemma 5 shows another necessary condition for the case in which $q_{out1}^*(w) > 0$.

Lemma 5. If $q_{out1}^*(w) > 0$, then $J_4(w, q_{\text{out }1}) < 0$, where

$$J_4(w, q_{\text{out }1}) := \inf\left\{G_1(w, q_{\text{out }1}) \,\middle|\, 0 < q_{out1} < F_1^{-1}\left(\frac{p-w}{p}\right),\right.$$
$$\left. 0 \le q_{\text{out }1} \le F_1^{-1}\left(\frac{p-c_s}{p}\right) - q_{\text{in }1}, c_s \le w \le c_b\right\}$$

Based on the above definitions, we can write the following two optimization problems to find the optimal potential stationary point of $\Pi_s(w, q_{out1}^*(w))$:

$$\max \Pi_s(w, q_{\text{out }1})$$
$$\text{s.t.} \begin{cases} p[1 - F_1(q_{\text{out }1})] - w - \frac{\mathrm{d}\alpha(q_{\text{out }1})}{\mathrm{d}q_{\text{out }1}}\pi_{b2}(w) = 0, \\ -pf_1(q_{\text{out }1}) - \frac{\mathrm{d}^2\alpha(q_{\text{out }1})}{\mathrm{d}q_{\text{out }1}^2}\pi_{b2}(w) \le 0, \\ G_1(w, q_{\text{out }1}) = 0, \\ G_2(w, q_{\text{out }1}) \le 0, \\ c_s \le w \le c_b \\ 0 \le q_{\text{out }1} \le F_2^{-1}\left(\frac{p-w}{p}\right) \end{cases} \tag{12}$$

for the case that $q_{in1}^* = 0$;

$$\max \Pi_s(w, q_{\text{out }1})$$
$$\text{s.t.} \begin{cases} c_b - w - \frac{\mathrm{d}\alpha(q_{\text{out }1})}{\mathrm{d}q_{\text{out}}}\pi_{b2}(w) = 0 \\ -pf_1\left(F_1^{-1}\left(\frac{p-c_b}{p}\right)\right) - \frac{\mathrm{d}^2\alpha(q_{\text{out }1})}{\mathrm{d}q_{\text{out }1}^2}\pi_{b2}(w) \le 0, \\ G_1(w, q_{\text{out }1}) = 0, \\ G_2(w, q_{\text{out }1}) \le 0, \\ c_s \le w \le c_b \\ 0 \le q_{\text{out }1} \le F_2^{-1}\left(\frac{p-w}{p}\right) \end{cases} \tag{13}$$

for the case that $q_{in1}^* \neq 0$.

Let (w_1, q_{out1_1}) and (w_2, q_{out1_2}) be the optimal solutions to Problems (12) and (13), respectively. Let $\Pi_s(\hat{w}, \hat{q}_{out1}) := \max\{\Pi_s(w_1, q_{out1_1}), \Pi_s(w_2, q_{out1_2})\}$, where $(\hat{w}, \hat{q}_{out1})$ is the corresponding optimal potential stationary point of $\Pi_s(w, q_{out1}^*(w))$. Note that if Problem (12) and Problem (13) are both infeasible, then ($\hat{w}, \hat{q}_{\text{out }1}$) does not exist. **Theorem 6** shows the key properties of ($\hat{w}, \hat{q}_{\text{out }1}$).

Theorem 6. (1) If $(\hat{w}, \hat{q}_{\text{out }1})$ exists and $\Pi_s(\hat{w}, \hat{q}_{\text{out }1}) \ge \Pi_s(c_s, q_{\text{out}}^*(c_s))$, then $\Pi_s(\hat{w}, \hat{q}_{\text{out }1})$ is an upper bound of Problem (11). Specifically, if either $(\hat{q}_{out1}, 0)$ given $\hat{\text{o}}\text{ut } 1 = q_{\text{out}1_1}$ or $\left(\hat{q}_{\text{out }1}, F_1^{-1}\left(\frac{p-c_b}{p}\right) - \hat{q}_{\text{out }1}\right)$ given $\hat{q}_{\text{out }1} = q_{\text{out }1_2}$ is optimal to the problem

$$\max_{q_{\text{out}1}, q_{\text{in }1} \ge 0} \Pi_b(\hat{w}, q_{\text{out }1}, q_{\text{in }1}),$$

then $w^* = \hat{w}, q_{\text{out }1}^*(w^*) = \hat{q}_{\text{out }1}, q_{\text{in }1}^*(w^*) = 0$ or $q_{\text{in }1}^*(w^*) = F_1^{-1}\left(\frac{p-c_b}{p}\right) - \hat{q}_{\text{out }1}$. (2) If ($\hat{w}, \hat{q}_{\text{out }1}$) does not exist, then one of the following two statements is true:

$$w^* = c_s, \left(q_{\text{out }1}^*(c_s), q_{\text{in }1}^*(c_s)\right) = \underset{q_{\text{out }1}, q_{\text{in }1} \in [0,1]}{\arg\max}\ \Pi_b(c_s, q_{\text{out }1}, q_{\text{in }1})$$

$w^*, q_{out1}^*(w^*)$ and $q_{\text{in }1}^*(w^*)$ are shown in Theorem 3.

Equilibrium $w^* = \hat{w}, q_{out1}^*(w^*) = \hat{q}_{\text{out}1}, q_{in1}^*(w^*) = 0$ or $q_{in1}^*(w^*) = F_1^{-1}\left(\frac{p-c_b}{p}\right) - \hat{q}_{\text{out }1}$ undoubtedly ideal for the supplier. However, $(\hat{w}, \hat{q}_{\text{out }1})$ is difficult to search because the constraints are non-convex, especially the third and the fourth constraints of Problem (12) or (13). Now, we modify the third and fourth constraints of Problems (12) and (13) to form two new problems:

$$\max \Pi_s(w, q_{\text{out }1})$$
$$\text{s.t.} \begin{cases} p[1-F_1(q_{\text{out }1})] - w - \frac{\mathrm{d}\alpha(q_{\text{out }1})}{\mathrm{d}q_{\text{out }1}}\pi_{b2}(w) = 0, \\ -pf_1(q_{\text{out }1}) - \frac{\mathrm{d}^2\alpha(q_{\text{out}})}{\mathrm{d}q_{\text{out }1}^2}\pi_{b2}(w) \le 0, \\ g_1(w, q_{\text{out }1}) = 0 \\ q_{\text{out }1} + (1-\alpha(q_{\text{out }1}))F_2^{-1}\left(\frac{p-w}{p}\right) - (w-c_s)\frac{1-\alpha(q_{\text{out }1})}{pf_2\left(\frac{p-w}{p}\right)} = 0, \\ J_5(w, q_{\text{out }1}) \ge 0, \\ c_s \le w \le c_b, \\ 0 \le q_{\text{out }1} \le F_2^{-1}\left(\frac{p-w}{p}\right). \end{cases} \tag{14}$$

where

$$J_5(w, q_{\text{out }1}) := 2 + (w-c_s)\frac{\mathrm{d}f_2\left(\frac{p-w}{p}\right)}{\mathrm{d}\left(\frac{p-w}{p}\right)}\frac{1}{pf_2\left(\frac{p-w}{p}\right)} + \left(\frac{\mathrm{d}\alpha(q_{\text{out }1})}{q_{\text{out }1}}\right)^2 \frac{\pi_{s2}}{(1-\alpha(q_{out1}))A_1(w, q_{out1})},$$
$$A_1(w, q_{\text{out }1}) = pf_1(q_{\text{out }1}) + \frac{\mathrm{d}^2\alpha(q_{out1})}{\mathrm{d}q_{\text{out }1}^2}\pi_b(w).$$

$$\max \Pi_s(w, q_{\text{out }1})$$
$$\text{s.t.} \begin{cases} c_b - w - \frac{\mathrm{d}\alpha(q_{\text{out }1})}{\mathrm{d}q_{\text{out }1}}\pi_{b2}(w) = 0 \\ -pf_1\left(F_1^{-1}\left(\frac{p-c_b}{p}\right)\right) - \frac{\mathrm{d}^2\alpha(q_{\text{out }1})}{\mathrm{d}q_{\text{out }1}^2}\pi_{b2}(w) \le 0, \\ g_1(w, q_{\text{out }1}) = 0, \\ q_{\text{out }1} + (1-\alpha(q_{\text{out }1}))F_2^{-1}\left(\frac{p-w}{p}\right) - (w-c_s)\frac{1-\alpha(q_{\text{out}})}{pf_2\left(\frac{p-w}{p}\right)} = 0, \\ J_6(w, q_{\text{out }1}) \ge 0, \\ c_s \le w \le c_b, \\ 0 \le q_{\text{out }1} \le F_2^{-1}\left(\frac{p-w}{p}\right). \end{cases} \tag{15}$$

where

$$J_6(w, q_{out1}) := 2 + (w-c_s)\frac{df_2\left(\frac{p-w}{p}\right)}{d\left(\frac{p-w}{p}\right)}\frac{1}{pf_2\left(\frac{p-w}{p}\right)} + \left(\frac{d\alpha(q_{out\ 1})}{q_{out\ 1}}\right)^2 \frac{\pi_{s2}}{(1-\alpha(q_{out}))A_2(w, q_{out1})},$$
$$A_2(w, q_{out\ 1}) = \frac{d^2\alpha(q_{out\ 1})}{d(q_{out\ 1})^2}\pi_b(w).$$

Let (w_3, q_{out1_3}) and (w_4, q_{out1_4}) be the optimal solutions to Problems (14) and (15), respectively. Let $\Pi_s(\tilde{w}, \tilde{q}_{\text{out }1}) := \max\{\Pi_s(w_3, q_{\text{out }1_3}), \Pi_s(w_4, q_{out1_4})\}$,
where $(\tilde{w}, \tilde{q}_{\text{out }1})$ is the corresponding optimal solution. Note that if Problem (14) and Problem (15) are both infeasible, then $(\tilde{w}, \tilde{q}_{out1})$ does not exist. Theorem 7 shows the key properties of $(\tilde{w}, \tilde{q}_{\text{out }1})$.

Theorem 7. If $(\tilde{w}, \tilde{q}_{out1})$ exists, then it is feasible for Problem (12) or (13). Specifically, if either $(\tilde{q}_{\text{out }1}, 0)$ given $\tilde{q}_{\text{out }1} = q_{\text{out }1_3}$ or $\left(\tilde{q}_{\text{out }1}, F_1^{-1}\left(\frac{p-c_b}{p}\right) - \tilde{q}_{\text{out }1}\right)$ given $\tilde{q}_{\text{out }1} = q_{out1_4}$ is optimal for the following problem

$$\max_{q_{\text{out }1}, q_{\text{in }1} \ge 0} \Pi_b(\tilde{w}, q_{\text{out }1}, q_{\text{in }1})$$

then $\Pi_s(\tilde{w}, \tilde{q}_{\text{out }1})$ is a lower bound of Problem (11).
Actually, $\tilde{w}$ is the optimal solution satisfying $\mathrm{d}q_{out1}^*(w)/\mathrm{d}w = 0$ and $\mathrm{d}^2q_{out1}^*(w)/\mathrm{d}w^2 < 0$. That is, $\tilde{w}$ is a local maximum point of $q_{out1}^*(w)$.

COMPUTATIONAL STUDIES

In this section, we conduct computational studies to illustrate the potential impacts of the findings above on outsourcing behaviors in the supply chain under uncertain demands. Additionally, we investigate the impact of product information on the overall outcome.

Let $\alpha(q_{out1}) = 2q_{out1} - q_{out\,1}^2$. For the convenience of calculation, we consider three demand distributions: a uniform distribution $U(0,1)$, high-demand distribution $F_H(\cdot)$ and high-demand distribution $F_L(\cdot)$, where the probability destiny functions F_H and F_L are defined as:

$$f_H(x) = \begin{cases} 0.5, & 0 < x \le 0.5 \\ 1.5, & 0.5 < x < 1 \end{cases}, \qquad f_L(x) = \begin{cases} 1.5, & 0 < x \le 0.5 \\ 0.5, & 0.5 < x < 1 \end{cases}$$

Stochastic demand is more likely to fall within the High-demand interval (0.5,1] in distribution $F_H(\cdot)$, while in distribution $F_L(\cdot)$, it is more likely to fall within the high-demand interval (0,0.5]. Therefore, based on the above three types of probability distribution, we define nine scenario types as shown in Table 2.

Table 2. Scenario types defined by demand probability distributions in two stages

	The probability distribution of D_1	The probability distribution of D_2
Type 1	$U(0,1)$	$U(0,1)$
Type 2	$U(0,1)$	$F_H(\cdot)$
Type 3	$U(0,1)$	$F_L(\cdot)$
Type 4	$F_H(\cdot)$	$U(0,1)$
Type 5	$F_L(\cdot)$	$U(0,1)$
Type 6	$F_H(\cdot)$	$F_L(\cdot)$
Type 7	$F_L(\cdot)$	$F_H(\cdot)$
Type 8	$F_H(\cdot)$	$F_H(\cdot)$
Type 9	$F_L(\cdot)$	$F_L(\cdot)$

Since Problem (10) and Problem (11) cannot be solved analytically, we design the following algorithm to find the approximate optimal solutions $(\breve{w}^*, \breve{q}^*_{out\,1}, \breve{q}^*_{in\,1}, \breve{q}^*_{out\,2}, \breve{q}^*_s)$ and optimal values $(\breve{\Pi}^*_b, \breve{\Pi}^*_s)$. All the equilibrium information mentioned hereinafter is denoted by $\breve{w}^*, \breve{q}^*_{out1}, \breve{q}^*_{in\,1}, \breve{q}^*_{out2}, \breve{q}^*_s, \breve{\Pi}^*_b$ and $\breve{\Pi}^*_s$.

Algorithm 1. Approximate **optimal solutions and optimal values of Problem (10) and Problem (11)**

Input: p, c_b, c_s
Output: $\breve{w}^*, \breve{q}^*_{out\,1}, \breve{q}^*_{in\,1}, \breve{q}^*_{out2}, \breve{q}^*_s, \breve{\Pi}^*_b, \breve{\Pi}^*_s$

1: $q^{in}_{out1} \leftarrow 0, q^{in}_{in1} \leftarrow F_1^{-1}\left(\frac{p - c_b}{p}\right), w^{in} \leftarrow \arg\max \Pi_s\left(w, q^{in}_{out1}\right)$ (via **Theorem 3**), $w \in [c_s, c_b]$
2: $\Pi^{in}_b \leftarrow \Pi_b\left(w^{in}, q^{in}_{out1}, q^{in}_{in1}\right), \Pi^{in}_s \leftarrow \Pi_s\left(w^{in}, q^{in}_{out1}\right)$
3: $\breve{w}^* \leftarrow w^{in}, \breve{q}^*_{out1} \leftarrow q^{in}_{out1}, \breve{q}^*_{in1} \leftarrow q^{in}_{in1}$
4: $\breve{\Pi}^*_b \leftarrow \Pi^{in}_b, \breve{\Pi}^*_s \leftarrow \Pi^{in}_s$
5: $q^{\max}_{in1} \leftarrow F_1^{-1}\left(\frac{p - c_b}{p}\right)$ (via **Theorem 2**)
6: $w^{\text{tem}} \leftarrow c_s$
7: **while** $w^{\text{tem}} \le c_b$ **do**
8: $\quad (q^{\text{tem}}_{out\,1}, q^{\text{tem}}_{in\,1}) \leftarrow \arg\max\{\Pi_b(w^{\text{tem}}, q_{out\,1}, q_{in\,1}) \mid 0 \le q_{out\,1} \le q^{\max}_{out1}, 0 \le q_{in\,1} \le q^{\max}_{in1}\}$
9: $\quad \Pi^{\text{tem}}_b \leftarrow \Pi_b(w^{\text{tem}}, q^{tem}_{out1}, q^{\text{tem}}_{in1}), \Pi^{\text{tem}}_s \leftarrow \Pi_s(w^{\text{tem}}, q^{\text{tem}}_{out1})$
10: $\quad$ **if** $\left(\Pi^{\text{tem}}_s \ge \breve{\Pi}^*_s\right.$ and $\left.\Pi^{\text{tem}}_b \ge \Pi^{in}_b\right)$ **then**
11: $\qquad \breve{w}^* \leftarrow w^{\text{tem}}, \breve{q}^*_{out1} \leftarrow q^{tem}_{out1}, \breve{q}^*_{in1} \leftarrow q^{\text{tem}}_{in1}$
12: $\qquad \breve{\Pi}^*_b \leftarrow \Pi^{tem}_b, \breve{\Pi}^*_s \leftarrow \Pi^{tem}_s$
13: $\qquad w^{\text{tem}} \leftarrow w^{tem} + 0.01$
14: $\breve{q}^*_{out2} \leftarrow \left(1 - \alpha(\breve{q}^*_{out1})\right)F_2^{-1}\left(\frac{p - \breve{w}^*}{p}\right), \breve{q}^*_s \leftarrow \alpha(\breve{q}^*_{out1})F_2^{-1}\left(\frac{p - c_s}{p}\right)$ (via **Lemma 1**)

The design idea of the Algorithm 1 is as follows:

In steps $1 - 5$, we initialize the optimal solutions and values. We first assume that production in-house only is the equilibrium, so the corresponding solutions $w^{in}, q^{in}_{out1}, q^{in}_{in}$ and values Π^{in}_b, Π^{in}_s are assigned to the optimal solutions and values. Moreover, the upper bound of $q_{in1}, q^{\max}_{in1}$, is determined via **Theorem 2;**

In steps 6 - 13, we enumerate $w^{tem} \in [c_s, c_b]$ with a step size of 0.01 to determine whether there are better solutions for Problem (10) and Problem (11). The condition of step 10 ensures that when the supplier finds a better w^{tem} to increase his total expected profit, the buyer will accept this outsourcing price (that is, the buyer's total expected profit based on w^{tem} is not lower than that when she refuses the outsourcing price, i.e., $\Pi_b^{tem} \geq \Pi_b^{in}$). After the loop, $\breve{w}^*, \breve{q}_{out1}^*, \breve{q}_{in1}^*, \breve{\Pi}_b^*, \breve{\Pi}_s^*$ are determined;

In step 14, $\breve{q}_{\text{out }2}^*$ and $\breve{q}_s^*$ are determined via **Lemma 1**.

Outsourcing Strategies under Different Scenarios

We first explore the equilibrium type in the different scenarios defined above. Let $p = 0.5, c_b = 0.2, c_s = 0.1$, and the approximate optimal solutions and values of the buyer and supplier in different scenarios are solved by MATLAB R2016a (the 'fmincon' function is used to solve the nonlinear optimization problem in Algorithm 4-1). Based on the approximate optimal solutions, we define the following three equilibrium types: O_1 refers to Outsourcing in Stage 1 with $w > c_s$ and $\breve{q}_{in}^* = 0$, i.e., buyer completely outsources in Stage 1 and ignores the value chain climbing of the supplier. O_2 refers to Outsourcing in Stage 1 with $w > c_s$ and $\breve{q}_{in}^* > 0$,i.e., buyer simultaneously outsources and produces in-house in Stage 1 to balance the benefits of cost reduction and the losses caused by value chain climbing. I refers to In-house Production in Stage 1 with $\breve{w}^* = c_b$. The results are shown in Table 3; "E.T." means equilibrium type.

Table 3. Equilibrium information under different scenarios given $p = 0.5, c_b = 0.2, c_s = 0.1$

	$\breve{w}^*$	$\breve{q}_{\text{out }1}^*$	$\breve{q}_{\text{in }1}^*$	$\breve{q}_{\text{out }2}^*$	$\breve{q}_s^*$	E.T.	Π_b^*	Π_s^*
Type 1	0.2000	0	0.6000	0.6007	0	I	0.1800	0.0600
Type 2	0.2000	0	0.6000	0.7333	0	I	0.2292	0.0556
Type 3	0.1200	0.6096	0	0.0792	0.5086	O_1	0.1534	0.0505
Type 4	0.2000	0	0.7333	0.6007	0	I	0.2294	0.0333
Type 5	0.1300	0.1548	0.0300	0.5287	0.2285	O_2	0.1513	0.0358
Type 6	0.1200	0.7847	0	0.0241	0.5722	O_1	0.2054	0.0583
Type 7	0.2000	0	0.4000	0.7333	0	I	0.1992	0.0556
Type 8	0.2000	0	0.7333	0.7333	0	I	0.2783	0.0556
Type 9	0.1300	0.3304	0	0.2212	0.3309	O_1	0.1222	0.0395

From Table 3, we can see that given parameters p, c_b, c_s, the equilibrium type is diverse under different scenario types and follows the following properties:

The equilibrium types Outsourcing in Stage 1(O_1 and O_2) appear in the scenarios where the stochastic demand follows the low-demand distribution $F_L(\cdot)$ at least in one stage (Stage 1 or Stage 2). In such scenarios, maintaining market share is not beneficial for the buyer because the demand is sluggish. Hence, the cost advantage driven by outsourcing becomes the critical path to improve the buyer's profit, and the supplier will always accept the outsourcing contract to climb the value chain. Specifically, when demand is likely to be low in Stage 2 (Type 3, Type 6, and Type 9), the buyer will not limit the supplier's value chain climbing $(O_1, \breve{q}_{in1}^* = 0)$ indicating that the increase of unit profit rate brought by cost advantage always overwhelms the reduction of the market share brought by vertical competition from the value chain climbing. Moreover, when the market expectation in Stage 2 is neither high nor low (Type 5), the buyer has to simultaneously consider the price advantage generated by outsourcing and the measures to control the market losses generated by the value chain climbing $(O_2, \breve{q}_{in1}^* = 0)$.

The equilibrium types In-house Production Only in Stage 1 (I) appear in the scenarios where no stochastic demand follows the low-demand distribution $F_L(\cdot)$ in both stages. In these scenarios, at least at a stage when the market may be very prosperous, the difference between the production cost of the buyer and the outsourcing price provided by the supplier is within the acceptable range. Therefore, monopolizing the market with higher production prices is more profitable than outsourcing to cut costs in Stage 2.

The Impact of Production Parameters on Outsourcing Strategies

Next, we study the impact of the production parameters, p, c_b, c_s, on the outsourcing strategies of the buyer and the supplier and their total expected profits in different scenarios. Specifically, we investigate the impacts of the following three factors: (1) the difference between p and c_b; (2) the difference between p and c_s; (3) the difference between c_b and c_s.

The difference between p and c_b signifies the buyer's profit margin when the buyer refrains from outsourcing in Stage 1; the difference between p and c_s represents the supplier's profit margin upon market entry; and the difference between c_b and c_s

indicates the extent of cost advantage gained by the buyer through outsourcing. We considered the two combinations: (1) fixed c_b and c_s, changed p; (2) fixed p and c_s, changed c_b. First, we set $c_b = 0.2, c_s = 0.1$, and p increases from 0.3 to 0.9 with a step size of 0.1. The equilibrium type and approximate total expected profits of t buyer and supplier are shown in Table 4, Figure 2 and Figure 3. Different from Table 3, there are more types of outsourcing equilibrium in Table 4.

Table 4. Equilibrium type under different market prices and scenarios

p	0.30	0.40	0.50	0.60	0.70	0.80	0.90
Type 1	O_1	O_1	I_2	I_2	I_2	I_2	I_2
Type 2	O_1	I_2	I_2	I_2	I_2	I_2	I_2
Type 3	O_1	O_1	O_1	I_2	I_2	I_1	I_1
Type 4	O_1	O_1	I_2	I_2	I_2	I_2	I_2
Type 5	O_1	O_2	O_2	I_2	I_2	I_2	I_2
Type 6	O_1	O_1	O_1	I_2	I_2	I_1	I_1
Type 7	O_1	I_2	I_2	I_2	I_2	I_2	I_2
Type 8	O_1	O_3	I_2	I_2	I_2	I_2	I_2
Type 9	O_1	O_1	O_1	O_3	I_2	I_1	I_1

O_1 refers to Outsourcing in Stage 1 with $w > c_s$ and $\check{q}_{\text{in}}^* = 0$;

O_2 refers to Outsourcing in Stage 1 with $w > c_s$ and $\check{q}_{in}^* > 0$;

O_3 refers to Outsourcing in Stage 1 with $w = c_s$ and $\check{q}_{in}^* = 0$;

I_1 refers to In-house Production in Stage 1, $\check{w}^* < c_b$;

I_2 refers to In-house Production in Stage 1, $\check{w}^* = c_b$.

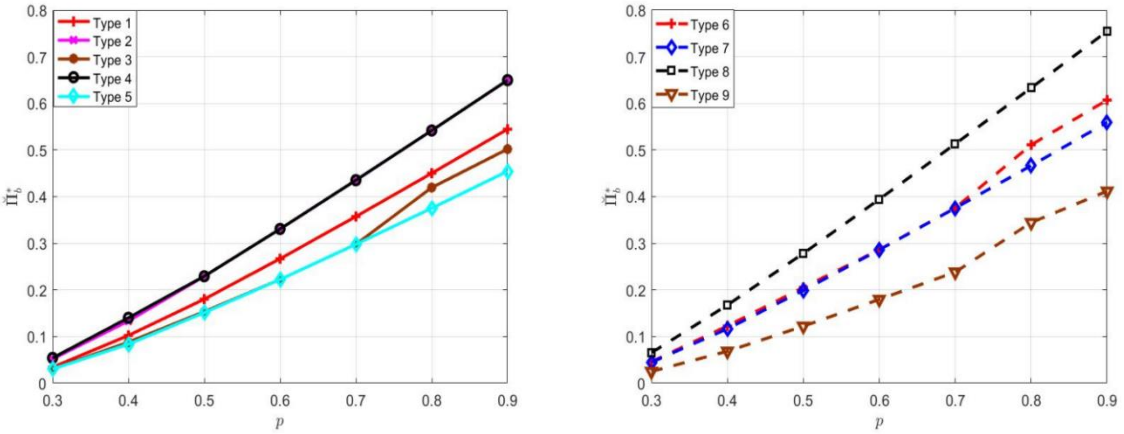
From Table 4 , we see that given c_b and c_s, there are six types of equilibrium evolution as p increases: $I_2, O_1 - I_2, O_1 - O_2 - I_2, O_1 - I_2 - I_1, O_1 - O_3 - I_2$, and $O_1 - O_3 - I_2 - I_1$. The interpretation of the change in equilibrium is as follows:

(1) For each scenario, when p is small enough, the profit margin of the buyer is too small if she produces in-house only in Stage 1. Here, she can obtain more benefits via cost reduction through outsourcing, so she is motivated to outsource to the supplier in both stages, even if the outsourcing price provided by the supplier is higher than his cost and she has to face competition from the supplier in the product market in Stage 2. Moreover, the supplier can profit from the buyer through outsourcing and profit in the product market through value chain climbing without restriction (Equilibrium Type O_1). As p increases, the profit margin of the buyer when she produces in-house only in Stage 1 is still small, so it is cost-effective for the buyer to reduce the cost through outsourcing. Nevertheless, in this situation, the buyer will limit the supplier's value chain climbing behavior by producing a part of the product quantity in Stage 1($\check{q}_{in1}^* > 0$). Meanwhile, the optimal outsourcing price $\check{w}^*$ exceeds his production costc_s, which means that he can obtain the outsourcing profit from the buyer. However, the buyer restricts his value chain climbing behavior to a certain extent (Equilibrium Type O_2). Alternatively, the supplier provides the lowest outsourcing price ($w = c_s$) in exchange for the buyer totally outsourcing ($\check{q}_{in1}^* = 0$). In this case, the supplier can only profit in the product market by value chain climbing in Stage 2 (Equilibrium type O_3). Compared to O_3, O_2 seems more common under different scenarios.

(2) For each scenario, when p exceeds a specific value, the cost advantage generated by outsourcing to the buyer cannot offset the seller's losses due to value chain climbing, so she will not outsource in Stage 1 to prevent the supplier from competing with her in Stage 2. When the supplier knows that the buyer prevents his value chain climbing behavior, he may provide the highest outsourcing price ($w = c_b$). In practice, this equilibrium represents when the buyer and supplier do not cooperate (Equilibrium I_2). Alternatively, the supplier will offer a lower outsourcing price than the buyer's production cost ($w < c_b$) to encourage the buyer to outsource in Stage 2 to obtain positive benefits, and the buyer can simultaneously profit through outsourcing and maintaining the entire share in Stage 2 (Equilibrium I_1). Note that in Table 4, the evolution of the equilibrium is from I_2 to I_1 as p increases. In most scenarios, the equilibrium I_2 is the end of the evolution process. However, in a few scenarios (Type 3, Type 6, and Type 9), due to the low demand distribution in Stage 2, the buyer tends to outsource to improve the profit margin of the product to offset the sluggish demand when p is large enough ($p \geq 0.80$), showing the transition from the equilibrium I_2 to Equilibrium I_1.

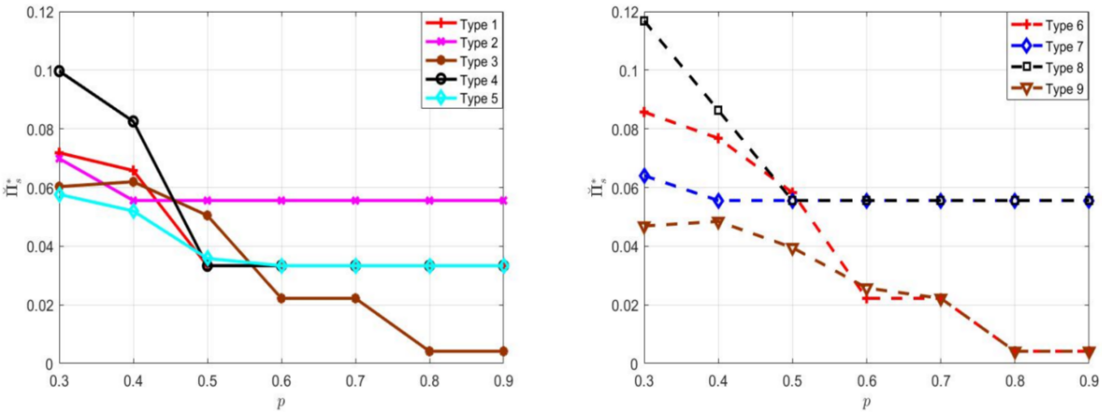
From Figures 2 and 3, we observe that $\breve{\Pi}_b^*$ increases as p increases, while $\breve{\Pi}_s^*$ shows three modes: decreasing-flat, increasing-decreasing-flat, or increasing-decreasing-flat-decreasing-flat with the increase of p, indicating a downward trend in general.

As for the different demand combinations, we find that the more the highdemand distribution ($F_H(\cdot)$) appears in a scenario, the higher $\breve{\Pi}_b^*$ is given each value of p, and the more the low-demand distribution ($F_L(\cdot)$) appears in a scenario, the smaller $\breve{\Pi}_b^*$ is given each value of p, which shows the market's high expected prosperity always positively impacts the buyer's profit. For the supplier, when $p \geq 0.60$, $\breve{\Pi}_s^*$ exhibits the same trend as $\breve{\Pi}_b^*$ given p, indicating that only when the unit price of the product is high enough, the expected market prosperity will bring more profits to the supplier. Specifically, $\breve{\Pi}_s^*$ is higher when $D_2 \sim F_H(\cdot)$ and $p \geq 0.5$.



2a: Type 1 to Type 5 2b: Type 6 to Type 9

Figure 2: The change of the $\breve{\Pi}_b^*$ as p increases under Type 1 to Type 9



3a: Type 1 to Type 5 3b: Type 6 to Type 9

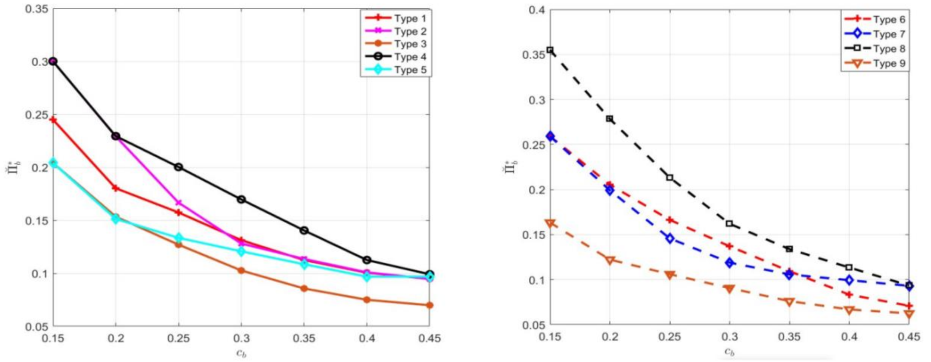
Figure 3: The change of the $\breve{\Pi}_s^*$ as p increases under Type 1 to Type 9

Then, we set $p = 0.5$, $c_s = 0.1$, and c_b increases from 0.15 to 0.45 with a step size of 0.05, the results are shown in Table 5, Figure 4 and Figure 5.

Table 5. Equilibrium type under different market prices and scenarios

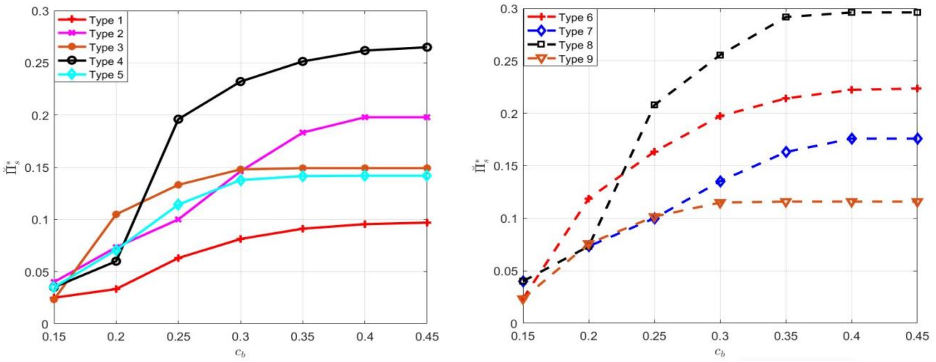
c_b	0.15	0.20	0.25	0.30	0.35	0.40	0.45
Type 1	I_2	I_2	O_1	O_1	O_1	O_1	O_1
Type 2	I_2	I_2	I_2	O_1	O_1	O_1	O_1
Type 3	I_2	O_1	O_1	O_1	O_1	O_1	O_1
Type 4	I_2	I_2	O_1	O_1	O_1	O_1	O_1
Type 5	I_2	O_1	O_1	O_1	O_1	O_1	O_1
Type 6	I_2	O_1	O_1	O_1	O_1	O_1	O_1
Type 7	I_2	I_2	I_1	O_1	O_1	O_1	O_1
Type 8	I_2	I_2	O_1	O_1	O_1	O_1	O_1
Type 9	I_2	O_1	O_1	O_1	O_1	O_1	O_1

Table 5 indicates that the equilibrium type shifts from I_2 to O_1 as c_b grows in every scenario except Type 7 , where it shifts from I_2 to I_1 to O_1. I_2 represents complete in-house production, and O_1 represents the exact opposite outsourcing decision in Stage 1, showing that when the cost of own production is low enough, the buyer will not outsource because the cost advantage generated by outsourcing is not apparent; however, once the cost of her production is high enough, outsourcing is an inevitable choice, regardless of the external market environment. Undeniably, in scenarios where the low demand distribution appears in at least one stage (Type 3, Type 5, Type 6 , and Type 9), buyers start outsourcing when $c_b = 0.20$. In contrast, in other scenarios, the buyer starts at $c_b = 0.25$ or $c_b = 0.30$. Interestingly, the buyer starts outsourcing when $c_b = 0.25$ in Type 8 with two high-demand distributions, and she starts outsourcing when $c_b = 0.30$ in Type 2 with only high-demand distribution in Stage 2. This comparison shows that when the expected market demands of a product are always high, the buyer may also outsource when the cost advantage is relatively small. Specifically, the supplier provides an outsourcing price lower than the buyer's production cost (Equilibrium type I_1) only when $c_b = 0.25$ in Type 7, showing that the outsourcing relationship exists in Stage 2. Thus, the buyer can obtain a cost advantage in Stage 2 while maintaining the entire market share, and the supplier profits from outsourcing.



4a: Type 1 to Type 5 4b: Type 6 to Type 9

Figure 4. The change of the $\breve{\Pi}_b^*$ as c_b increases under Type 1 to Type 9



5a: Type 1 to Type 5 5b: Type 6 to Type 9

Figure 5. The change of the $\breve{\Pi}_s^*$ as c_b increases under Type 1 to Type 9

From Figure 4 and Figure 5, we can see that $\breve{\Pi}_b^*$ decreases as c_b increases, while $\breve{\Pi}_s^*$ follows the opposite trend. Similar to Figure 2 and Figure 3, the more the high-demand distribution ($F_H(\cdot)$) appears in a scenario, the higher $\breve{\Pi}_b^*$ is given each value of c_b. Correspondingly, the more the low-demand distribution ($F_L(\cdot)$) appears in a scenario, the smaller $\breve{\Pi}_b^*$ is given each value of c_b. As for the supplier, $\breve{\Pi}_s^*$ follows the same trend when $c_b \geq 0.25$. Specifically, $\breve{\Pi}_s^*$ is higher when $D_1 \sim F_H(\cdot)$ and $c_b \geq 0.25$.

CONCLUSION

Outsourcing supply chains have become increasingly competitive due to the value chain climbing phenomenon. Our study examines this impact in the context of demand uncertainty and quantity competition. The decision making process involves two stages. In Stage 1, the supplier determines the outsourcing price, and the buyer decides whether to accept the price and the quantities of in-house production and outsourcing. During this stage, the supplier learns from outsourcing to prepare for entering the product market in the next stage. In Stage 2, the supplier can enter the market to compete with the buyer if an outsourcing contract is in place from Stage 1. The buyer will outsource to the supplier because the product's life cycle expires at this stage. The demand in both stages, represented by D_1 and D_2, is stochastic.

We propose a basic two-stage game theory model in which the supplier sets the outsourcing price first and the buyer then investigates outsourcing strategies; the results of the theoretical analysis show the conditions under which the buyer and the supplier adopt different outsourcing strategies: complete production in-house in Stage 1, complete outsourcing in Stage 1, and partial outsourcing in Stage 1. Note that complete production in-house in Stage 1 is rare in many related studies. The properties of each equilibrium are also given. Then, we extend the basic model to the case in which the outsourcing cost the supplier provides is lower than the supplier's production cost. We identify the sufficient conditions under which this case occurs. The numerical study illustrates (i) the properties of the optimal outsourcing strategies of the supply chain and the corresponding total expected profits and (ii) the impact of demand uncertainty and other vital parameters on outsourcing strategies. We have found that it is more advantageous for the buyer to maintain the whole market by not outsourcing in Stage 1 when the production cost is relatively high. As the price of the product increases, buyers tend to shift from outsourcing to in-house production, while the opposite is true when the buyer's production cost increases. The market's stochastic parameters also highly impact the buyer and supplier's outsourcing behaviors.

Our exploration contributes to the supply chain literature's strategy-level study on competition relationships and operation-level study on outsourcing contracts. We show that complete outsourcing, partial outsourcing, and complete in-house production are proper for the outsourcing buyer depending on the parameters of stochastic demand when the supplier is relatively powerful.

We acknowledge that our work has limitations and propose two research directions for future investigations. Firstly, it is crucial to consider the buyer's learning effect to decrease production costs and enhance the ability to mitigate value chain climbing. Secondly, there is a high risk of supply disruption due to certain macroeconomic factors. Although these topics are not covered in this research, they are still essential and should be explored in future investigations.

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APPENDIX: PROOFS OF THEOREMS AND LEMMAS

1. Proof of Lemma 1

(1) If $q_{\text{out }1} = 0$, then $\alpha(q_{out1}) = \alpha(0) = 0$. From Equations (3)-(4), the best responses of the buyer and supplier in Stage 2 are $q_{\text{out }2}(w, q_{\text{out }1}) = q_{\text{out }2}(w, 0) = F_2^{-1}\left(\frac{p-w}{p}\right)$ and $q_s(q_{out1}) = q_s(0) = 0$, respectively, Lemma 1 holds. (2) If $q_{out1} > 0$, then for the buyer,

$$\frac{\partial \Pi_b(w, q_{out1}, q_{\text{in }1}, q_{out2})}{\partial q_{out}2} = \frac{\partial \pi_b^2(w, q_{out1}, q_{out2})}{\partial q_{out}2} = p\left[1 - F_2\left(\frac{q_{out2}}{1-\alpha(q_{out1})}\right)\right] - w$$

$$\frac{\partial^2 \Pi_b(w, q_{out1}, q_{\text{in }1}, q_{out2})}{\partial q_{out}^2} = \frac{\partial^2 \pi_b^2(w, q_{out}, q_{out})}{\partial q_{out}2} = -\frac{pf_2\left(\frac{q_{out}2}{1-\alpha(q_{out}1)}\right)}{1-\alpha(q_{out1})} < 0$$

Thus, $\Pi_b(w, q_{out1}, q_{\text{in }1}, q_{out2})$ is concave in q_{out2} for given w, q_{out1}, q_{in1}. Let $\frac{\partial \Pi_b(w, q_{\text{out }1}, q_{\text{in }1}, q_{\text{out }2})}{\partial q_{\text{out}}} = 0$, so the best response of the buyer in Stage 2 is

$$q_{out2}^*(w, q_{out1}) = \left(1-\alpha(q_{\text{out }1})\right)F_2^{-1}\left(\frac{p-w}{p}\right)$$

Similarly, for the supplier,

$$\frac{\partial \Pi_s(w, q_{\text{out }1}, q_{out2}, q_s)}{\partial q_s} = \frac{\partial \pi_s^2(w, q_{out}, q_{\text{out }2}, q_s)}{\partial q_s} = p\left[1 - F_2\left(\frac{q_s}{\alpha(q_{out1})}\right)\right] - c_s$$

$$\frac{\partial^2 \Pi_s(w, q_{out}, q_{out}, q_s)}{\partial q_s^2} = \frac{\partial^2 \pi_s^2(w, q_{out1}, q_{out2}, q_s)}{\partial q_s^2} = -\frac{pf_2\left(\frac{q_s}{\alpha(q_s)}\right)}{\alpha(q_{out1})} < 0$$

and the best response of the supplier in Stage 2 is

$$q_s^*(q_{\text{out }1}) = \alpha(q_{out1})F_2^{-1}\left(\frac{p-c_s}{p}\right)$$

2. Proof of Theorem 2

Take the first and second derivatives of $q_{\text{out }1}$ and $q_{\text{in}1}$; we have

$$\frac{\partial \Pi_b(w, q_{\text{out }1}, q_{\text{in }1})}{\partial q_{\text{out }1}} = p[1 - F_1(q_{\text{out }1} + q_{\text{in }1})] - w - \frac{d\alpha(q_{\text{out }1})}{dq_{\text{out }1}}\pi_{b2}(w)$$

$$\frac{\partial \Pi_b(w, q_{\text{out }1}, q_{\text{in }1})}{\partial q_{\text{in }1}} = p[1 - F_1(q_{\text{out }1} + q_{\text{in1 }1})] - c_b$$

$$\frac{\partial^2 \Pi_b(w, q_{\text{out }1}, q_{\text{in }1})}{\partial q_{\text{out }1}^2} = -pf_1(q_{\text{out }1} + q_{\text{in }1}) - \frac{d^2\alpha(q_{\text{out }1})}{dq_{\text{out }1}^2}\pi_{b2}(w)$$

$$\frac{\partial^2 \Pi_b(w, q_{\text{out }1}, q_{\text{in }1})}{\partial q_{\text{in }1}^2} = -pf_1(q_{\text{out }1} + q_{\text{in }1}) < 0$$

$$\frac{\partial^2 \Pi_b(w, q_{\text{out }1}, q_{\text{in }1})}{\partial q_{\text{out }1}\partial q_{\text{in }1}} = -pf_1(q_{\text{out }1} + q_{\text{in }1}) < 0$$

(a) If $q_{\text{out }1} \geq F_1^{-1}\left(\frac{p-w}{p}\right)$, then $F_1(q_{\text{out }1} + q_{in1}) \geq \frac{p-w}{p}$ based on the non-negativity of $q_{\text{in }1}$, and $\frac{\partial \Pi_b(w, q_{out1}, q_{in1})}{\partial q_{out1}} < 0$, so $\Pi_b(w, q_{\text{out }1}, q_{\text{in }1})$ is monotonically decreasing in $q_{\text{out }1} \in \left[F_1^{-1}\left(\frac{p-w}{p}\right), 1\right]$, $0 \leq q_{out1}^*(w) < F_1^{-1}\left(\frac{p-w}{p}\right)$. Similarly, we can prove that $0 \leq q_{in1}^*(w) \leq F_1^{-1}\left(\frac{p-c_b}{p}\right)$.

If $q^*_{out1}(w) + q^*_{in1}(w) \geq F_1^{-1}\left(\frac{p-c_s}{p}\right)$, then

$$\left.\frac{\partial \Pi_b(w, q_{out1}, q_{in1})}{\partial q_{out1}}\right|_{0<q_{out1}+q_{in1}\leq F_1^{-1}\left(\frac{p-c_s}{p}\right)} \leq c_s - w - \frac{\mathrm{d}\alpha(q_{out1})}{\mathrm{d}q_{out1}}\pi_{b2}(w) < 0$$

$$\left.\frac{\partial \Pi_b(w, q_{out1}, q_{\text{in }1})}{\partial q_{\text{out }1}}\right|_{0<q_{out1}+q_{\text{in }1}\leq F_1^{-1}\left(\frac{p-c_s}{p}\right)} \leq c_s - c_b < 0$$

That is, the buyer can increase her total expected profit by lowing $q_{\text{out }1}$ or $q_{\text{in }1}$, $q^*_{out1}(w) + q^*_{\text{in }1}(w) \leq F_1^{-1}\left(\frac{p-c_s}{p}\right)$.

(b) Because

$$\frac{\partial^3 \Pi_b(w, q_{\text{out }1}, q_{\text{in }1})}{\partial q^2_{\text{out }1}\partial w} = -\frac{\mathrm{d}^2\alpha(q_{\text{out }1})}{\mathrm{d}q^2_{\text{out }1}} p \frac{\mathrm{d}\pi_{b2}}{\mathrm{d}w} = -\frac{\mathrm{d}^2\alpha(q_{\text{out }1})}{\mathrm{d}q^2_{\text{out }1}} p\left(-F_2^{-1}\left(\frac{p-w}{p}\right)\right) < 0$$

we have that

$$\frac{\partial^2 \Pi_b(c_b, q_{out1}, q_{in1})}{\partial q^2_{out1}} = -pf_1(q_{out1} + q_{in1}) - \frac{\mathrm{d}^2\alpha(q_{out1})}{\mathrm{d}q^2_{\text{out }1}}\pi_{b2}(c_b)$$
$$< \frac{\partial^2 \Pi_b(w, q_{out1}, q_{\text{in }1})}{\partial q^2_{out1}} < \frac{\partial^2 \Pi_b(c_s, q_{out1}, q_{in1})}{\partial q^2_{out1}}$$

We rewrite J_1 as

$$J_1(q_{out1}, q_{\text{in }1}) = \inf\left\{\left.\frac{\partial^2 \Pi_b(c_b, q_{out1}, q_{\text{in }1})}{\partial q^o_{out1}}\right| 0 < q_{out1} < F_1^{-1}\left(\frac{p-w}{p}\right)\right.$$
$$\left. 0 \leq q_{\text{in }1} < F_1^{-1}\left(\frac{p-c_b}{p}\right), 0 \leq q_{out1} + q_{\text{in }1} \leq F_1^{-1}\left(\frac{p-c_s}{p}\right), c_s \leq w \leq c_b\right\}$$

Therefore, in the potential optimal solution interval shown in Theorem 2(a),

$$\frac{\partial^2 \Pi_b(w, q_{out1}, q_{in1})}{\partial q^2_{out1}} > J_1(q_{out1}, q_{in1}) \geq 0$$

$\Pi_b(w, q_{out1}, q_{in1})$ is convex in q_{out1}, and

$$\left.\frac{\partial \Pi_b(w, q_{out}, q_{\text{in }1})}{\partial q_{out}1}\right|_{0\leq q_{out1}\leq F_1^{-1}\left(\frac{p-w}{p}\right)} \leq \left.\frac{\partial \Pi_b(w, q_{out}, q_{in1})}{\partial q_{out}}\right|_{q_{out}=F_1^{-1}\left(\frac{p-w}{p}\right)} < 0$$

then $w \in [c_s, c_b]$, $q_{\text{in }1} \in \left[0, F_1^{-1}\left(\frac{p-c_b}{p}\right)\right]$, $\Pi_b(w, 0, q_{\text{in }1}) > \Pi_b(w, q_{\text{out }1}, q_{\text{in }1}), \forall q_{\text{out }1} \in \left[0, F_1^{-1}\left(\frac{p-w}{p}\right)\right]$. As a result, $q^*_{out1}(w) = 0, q^*_{in1}(w) > 0$.

3. Proof of Theorem 3

If $q^*_{out1}(w) = 0$, then $\alpha(q^*_{out1}(w)) = 0$. According to Lemma , $q^*_{out2}(w, 0) = F_2^{-1}\left(\frac{p-w^*}{p}\right)$, $q^*_s(0) = 0$. Then, Problem (10) becomes

$$\max_{q_{in1}\geq 0} \Pi_b(w, 0, q_{in1}) = \pi_b^1(w, 0, q_{in1}) + \pi_b^2(w, 0, q^*_{out2}(w, 0))$$

Take the first and second derivatives,

$$\frac{\partial \Pi_b(w, 0, q_{in1})}{\partial q_{in1}} = \frac{\partial \pi_b^1(w, 0, q_{in1})}{\partial q_{in1}} = p[1 - F_1(q_{in1})] - c_b$$

$$\frac{\partial^2 \Pi_b(w, 0, q_{in1})}{\partial q^2_{in1}} == -pf_1(q_{in1}) < 0$$

thus $q^*_{in1}(w)$ is independent of w and $q^*_{in1}(w) = q^*_{in1}(w^*) = F_1^{-1}\left(\frac{p-c_b}{p}\right)$.

If $q^*_{out1}(w) = 0$, then Problem (11) becomes

$$\max_{c_s \le w \le c_b} \Pi_s(w,0) = (w - c_s)F_1^{-1}\left(\frac{p-w}{p}\right) \qquad (A.1)$$

then

$$\frac{d\Pi_s(w,0)}{dw} = F_2^{-1}\left(\frac{p-w}{p}\right) - \frac{w - c_s}{pf_2\left(\frac{p-w}{p}\right)}$$

Therefore, $d\Pi_s(w,0)/dw < (\ge)0 \Leftrightarrow g(w) < (\ge)0$. Let $t(w) := \frac{p-w}{p}$,

$$\frac{d^2\Pi_s(w,0)}{dw^2} = \frac{1}{pf_2(t(w))}\left[-2 - \frac{(w-c_s)}{pf_2(t(w))}\frac{df_2(t(w))}{dt(w)}\right]$$

(a) Obviously, $\Pi_s(w,0) > \Pi_s(c_s,0) = 0\ \forall w \in (c_s, c_b]$, so $w^* > c_s$.

(b) If $F_2(\cdot)$ is an IFR and $0 < t(w) < \frac{p-c_s}{p} < 1$, then

$$r_2(t(w)) \le r_2\left(\frac{p-c_s}{p}\right) \le \frac{2p}{c_b - c_s}$$

then

$$\frac{df_2(t(w))}{dt(w)} \ge -f_2(t(w))r_2(t(w)) \ge -\frac{2pf_2(t(w))}{c_b - c_s}$$
$$\Rightarrow \frac{d^2\Pi_s(w,0)}{dw^2} \le \frac{2}{pf_2(t(w))}\left[-1 + \frac{w - c_s}{c_b - c_s}\right] \le 0.$$

That is, $\Pi_s(w,0)$ is concave in $w \in [c_s, c_b]$, so w^* is unique. We know that $g(c_s) > 0$, thus (1) $g(w) = 0, w \in (c_s, c_b)$ has an unique solution when $g(c_b) < 0$, and this solution is globally optimal; (2) $g(w) > g(c_b) \ge 0$, which means that $w^* = c_b$; (c) If the condition for (a) does not hold, the point satisfying $g(w) = 0$ (stationary point) and boundary point $w = c_b$ are still potential optimal solutions to Problem (A.1) because $\Pi_s(w,0)$ is continuous in w; searching over them will find w^*.

4. Proof of Lemma 4

Given $q_{in1}^*(w) = 0$, from the above results, we know that $0 < q_{out1}^*(w) < F_1^{-1}\left(\frac{p-w}{p}\right) < F_1^{-1}\left(\frac{p-c_s}{p}\right)$. Thus, when $c_s \le w \le c_b$, if $J_2(q_{\text{out }1}) = \sup\{-pf_1(q_{\text{out }1}) - \frac{d^2\alpha(q_{\text{out }1})}{dq_{\text{out }1}^o}\pi_{b2}(c_s) \Big| \ 0 < q_{\text{out }1} \le F_1^{-1}\left(\frac{p-c_s}{p}\right)\Big\} \le 0$, then when $0 < q_{\text{out }1} \le F_1^{-1}\left(\frac{p-c_s}{p}\right)$,

$$\frac{\partial^2\Pi_b(w, q_{out1}, 0)}{\partial q_{out1}^2} = -pf_1(q_{out1}) - \frac{d^2\alpha(q_{out1})}{dq_{out1}^2}\pi_{b2}(w)$$
$$\le -pf_1(q_{out1}) - \frac{d^2\alpha(q_{out1})}{dq_{out}^2 1}\pi_{b2}(c_s) < J_2(q_{out1}) \le 0$$

$\Pi_b(w, q_{\text{out }1}, 0)$ is concave in $q_{\text{out }1} \in \left(0, \frac{p-w}{p}\right)$. The first inequality holds because

$$\frac{d\pi_{b2}(w)}{dw} = -F_2^{-1}\left(\frac{p-w}{p}\right) < 0, \forall w \in [c_s, c_b],$$

and $d^2\alpha(q_{\text{out }1})/dq_{\text{out }1}^2 < 0$.

Given the concavity of $\Pi_b(w, q_{out1}, 0)$ in $q_{\text{out }1} \in \left(0, \frac{p-w}{p}\right)$, if $J_3(w, q_{out1}) = \inf\left\{p - w - \frac{d\alpha(q_{out1})}{dq_{out1}}\pi_{b2}(w) \Big| \ q_{\text{out }1} = 0, c_s \le w \le c_b\right\} > 0$, then we can show that the equation

$$\frac{\partial\Pi_b(w, q_{out1}, 0)}{\partial q_{out1}} = p[1 - F_1(q_{out1})] - w - \frac{d\alpha(q_{out1})}{dq_{out1})}\pi_{b2}(w) = 0$$

has a unique solution by contradiction. Suppose that $\exists w \in [c_s, c_b]$ satisfying

$$\frac{\partial\Pi_b(w, q_{\text{out }1}, 0)}{\partial q_{\text{out }1}} < 0 < J_3(w, q_{\text{out }1}) = \frac{\partial\Pi_b(w, q_{\text{out }1}, 0)}{\partial q_{\text{out }1}}\Bigg|_{q_{out1}=0, c_s \le w \le c_b},$$

then $\Pi_b(w, q_{out1}, 0)$ is monotonically decreasing in $q_{out1} \in \left(0, \frac{p-w}{p}\right), q_{out1}^*(w) = 0$, which is impossible.

5. Proof of Lemma 5

Take the first and second derivatives of $\Pi_s(w, q_{out1}^*(w))$, and define $\pi_{s2} := pE\left[\min\left(F_2^{-1}\left(\frac{p-c_s}{p}\right), D_2\right)\right] - c_s F_2^{-1}\left(\frac{p-c_s}{p}\right)$; we have

$$\begin{aligned}\frac{\mathrm{d}\Pi_s(w, q_{out1}^*(w))}{\mathrm{d}w} =& q_{out1}^*(w) + \left(1-\alpha(q_{out1}^*(w))\right)F_2^{-1}\left(\frac{p-w}{p}\right) \\ &+ \frac{\mathrm{d}q_{out1}^*(w)}{w}\left[(w-c_s)\left(1-\frac{\mathrm{d}\alpha(q_{out1}^*(w))}{\mathrm{d}q_{out1}^*(w)}F_2^{-1}\left(\frac{p-w}{p}\right)\right)\right. \\ &\left.+\frac{\mathrm{d}\alpha(q_{out1}^*(w))}{\mathrm{d}q_{out1}^*(w)}\pi_{s2}\right] - (w-c_s)\frac{1-\alpha(q_{out1}^*(w))}{pf_2\left(\frac{p-w}{p}\right)}\end{aligned}$$

$$\begin{aligned}\frac{\mathrm{d}^2\Pi_s(w, q_{out1}^*(w))}{\mathrm{d}w^2} =& 2\frac{\mathrm{d}q_{out1}^*(w)}{w}\left[1-\frac{\mathrm{d}\alpha(q_{out1}^*(w))}{\mathrm{d}q_{out1}^*(w)}F_2^{-1}\left(\frac{p-w}{p}\right)\right] - 2\frac{1-\alpha(q_{out1}^*(w))}{pf_2\left(\frac{p-w}{p}\right)} \\ &+ \frac{\mathrm{d}^2q_{out1}^*(w)}{w^2}\left[(w-c_s)\left(1-\frac{\mathrm{d}\alpha(q_{out1}^*(w))}{\mathrm{d}q_{out1}^*(w)}F_2^{-1}\left(\frac{p-w}{p}\right)\right)\right. \\ &\left.+\frac{\mathrm{d}\alpha(q_{out1}^*(w))}{\mathrm{d}q_{out1}^*(w)}\pi_{s2}\right] \\ &+ \frac{\mathrm{d}q_{out1}^*(w)}{w}\left[(w-c_s)\left(-\frac{\mathrm{d}^2\alpha(q_{out1}^*(w))}{\mathrm{d}(q_{out1}^*(w))^2}\frac{\mathrm{d}q_{out1}^*(w)}{w}F_2^{-1}\left(\frac{p-w}{p}\right)\right.\right. \\ &\left.\left.+\frac{\mathrm{d}\alpha(q_{out1}^*(w))}{\mathrm{d}(q_{out1}^*(w))}\frac{1}{pf_2\left(\frac{p-w}{p}\right)}\right) - \frac{\mathrm{d}^2\alpha(q_{out1}^*(w))}{\mathrm{d}(q_{out1}^*(w))^2}\frac{\mathrm{d}q_{out1}^*(w)}{w}\pi_{s2}\right] \\ &-(w-c_s)\left[-\frac{\mathrm{d}\alpha(q_{out1}^*(w))}{\mathrm{d}(q_{out1}^*(w))}\frac{\mathrm{d}q_{out1}^*(w)}{\mathrm{d}w}\frac{1}{pf_2\left(\frac{p-w}{p}\right)}\right. \\ &\left.+\frac{1-\alpha(q_{out1}^*(w))}{\left(pf_2\left(\frac{p\text{-w}}{p}\right)\right)^2}\frac{\mathrm{d}\left(\frac{p-w}{p}\right)}{\mathrm{d}\left(\frac{p-w}{p}\right)}\right]\end{aligned}$$

where $\mathrm{d}q_{out1}^*(w)/\mathrm{d}w$ is determined by the equation

$$\frac{\partial\Pi_b(w, q_{out1}, q_{\text{in }1})}{\partial q_{out1}} = 0 \Leftrightarrow \begin{cases} p[1-F_1(q_{out1})] - w - \dfrac{\mathrm{d}\alpha(q_{out1})}{\mathrm{d}q_{out1}}\pi_{b2}(w) = 0, & \text{if } q_{\text{in }1} = 0 \\ c_b - w - \dfrac{\mathrm{d}(q_{out1})}{\mathrm{d}q_{out\,1}}\pi_{b2}(w) = 0, & \text{if } q_{\text{in }1} > 0 \end{cases}$$

Thus,

$$\frac{\mathrm{d}q_{out1}^*(w)}{\mathrm{d}w}=\frac{A(w,q_{out1}^*(w))}{B(w,q_{out1}^*(w))},$$
$$A(w,q_{out1}^*(w)):=\frac{\mathrm{d}\alpha(q_{out1}^*(w))}{\mathrm{d}(q_{out1}^*(w))}F_2^{-1}\left(\frac{p-w}{p}\right)-1,$$
$$B(w,q_{out1}^*(w)):=\begin{cases}pf_1(q_{out1}^*(w))+\dfrac{\mathrm{d}^2\alpha\left(q_{out1}^*(w)\right)}{\mathrm{d}\left(q_{out1}^*(w)\right)^2}\pi_{b2}(w),\text{if } q_{\mathrm{in}\,1}^*(w)=0,\\ \dfrac{\mathrm{d}^2\alpha\left(q_{out1}^*(w)\right)}{\mathrm{d}\left(q_{out1}^*(w)\right)^2}\pi_{b2}(w),\text{if } q_{\mathrm{in}\,1}^*(w)>0\end{cases}$$
$$\frac{\mathrm{d}^2q_{out1}^*(w)}{\mathrm{d}w^2}=\left[\frac{\mathrm{d}^2\alpha(q_{out1}^*(w))}{\mathrm{d}(q_{out1}^*(w))^2}\frac{\mathrm{d}q_{out1}^*(w)}{\mathrm{d}w}F_2^{-1}\left(\frac{p-w}{p}\right)-\frac{\mathrm{d}\alpha(q_{out1}^*(w))}{\mathrm{d}(q_{out1}^*(w))}\frac{1}{pf_2\left(\frac{p-w}{p}\right)}\right]/B(w,q_{out1}^*(w))-\frac{A(w,q_{out1}^*(w))C(w,q_{out1}^*(w))}{[B(w,q_{out1}^*(w))]^2},$$
$$C(w,q_{out1}^*(w)):=\begin{cases}p\dfrac{\mathrm{d}f_1(q_{out1}^*(w))}{\mathrm{d}q_{out1}^*(w)}\dfrac{\mathrm{d}q_{out1}^*(w)}{\mathrm{d}w}+\dfrac{\mathrm{d}^3f_1(q_{out1}^*(w))}{\mathrm{d}(q_{out1}^*(w))^3}\dfrac{\mathrm{d}q_{out1}^*(w)}{\mathrm{d}w}\pi_{b2}(w)+\dfrac{\mathrm{d}^2f_1(q_{out1}^*(w))}{\mathrm{d}(q_{out1}^*(w))^2}\dfrac{\mathrm{d}\pi_{b2}(w)}{\mathrm{d}w}, & \text{if } q_{\mathrm{in}\,1}^*(w)=0\\ \dfrac{\mathrm{d}^3f_1(q_{out1}^*(w))}{\mathrm{d}(q_{out1}^*(w))^3}\dfrac{\mathrm{d}q_{out1}^*(w)}{\mathrm{d}w}\pi_{b2}(w)+\dfrac{\mathrm{d}^2f_1(q_{out1}^*(w))}{\mathrm{d}(q_{out1}^*(w))^2}\dfrac{\mathrm{d}\pi_{b2}(w)}{\mathrm{d}w}, & \text{if } q_{\mathrm{in}\,1}^*(w)>0\end{cases}$$

We prove the Lemma 5 by contradiction. Given $q_{out1}^*(w)>0$, we suppose that

$$J_4(w,q_{out1})=\inf\left\{G_1(w,q_{\mathrm{out}\,1})\,\middle|\,0<q_{out1}<F_1^{-1}\left(\frac{p-w}{p}\right),0\le q_{\mathrm{out}\,1}\le F_1^{-1}\left(\frac{p-c_s}{p}\right)-q_{\mathrm{in}\,1},c_s\le w\le c_b\right\}\ge 0$$

then

$$\frac{\mathrm{d}\Pi_s(w,q_{out1}^*(w))}{\mathrm{d}w}=\left\{G_1(w,q_{\mathrm{out}\,1})\,\middle|\,0<q_{out1}<F_1^{-1}\left(\frac{p-w}{p}\right),0\le q_{\mathrm{out}\,1}\le F_1^{-1}\left(\frac{p-c_s}{p}\right)-q_{\mathrm{in}\,1},c_s\le w\le c_b\right\}>J_4(w,q_{\mathrm{out}\,1})\ge 0,\forall w\in[c_s,c_b]$$

That is, $w^*=c_b$. However, we know that if $w^*=c_b$, then $q_{out1}^*(c_b)=0$, which contradicts the premise.

6. Proof of Theorem 6

1) For Problem (12), the first two constraints are the first and second partial derivatives of $\Pi_b(w,q_{out1},0)$ to q_{out1}, which form the feasible region of $(w,q_{out1}^*(w))$ given $q_{in1}^*(w)=0$. The middle two constraints contain the set of local maxima of Problem (11): $\{\mathrm{d}\Pi_s(w,q_{out1}^*(w))/\mathrm{d}w=0,\ \mathrm{d}^2\Pi_s(w,q_{out1}^*(w))/\mathrm{d}w^2\le 0\}$. The last two constraints are the regions of decision variables. Thus, the feasible region of Problem (12) contains the region $\Omega_1:=S_1\times M_1$, where S_1 is the set of local maxima of Problem (11) given $q_{in1}^*(w)=0$ and $M_1:=\{q_{out1}^*(w)\mid w\in S_1\}$. If Ω_1 is nonempty, then we define $\left(w_1',q_{out1_1}'\right):=\arg\max\{\Pi_s(w,q_{\mathrm{out}\,1})\mid w\in S_1,q_{\mathrm{out}\,1}\in M_1\}$. Obviously, $\Pi\left(w_1,q_{out1_1}\right)\ge\Pi\left(w_1',q_{out1_1}'\right)$.

For Problem (13), the first two constraints form the feasible region of $(w,q_{out1}^*(w))$ given $q_{in1}^*(w)=F_1^{-1}\left(\frac{p-c_b}{p}\right)-q_{out1}^*(w)$. The last four constraints are the same as those in Problem (12). Thus, the feasible region of Problem (13) contains the region $\Omega_2:=S_2\times M_2$, where S_2 is the set of local maxima of Problem (11) given $q_{\mathrm{in}\,1}^*(w)=F_1^{-1}\left(\frac{p-c_b}{p}\right)-q_{\mathrm{out}\,1}^*(w)$ and $M_2:=\{q_{\mathrm{out}\,1}^*(w)\mid w\in S_2\}$. If Ω_2 is nonempty, then we define $\left(w_2',q_{out1_2}'\right):=\arg\max\{\Pi_s(w,q_{out1})\mid w\in S_2,q_{\mathrm{out}\,1}\in M_2\}$. Obviously, $\Pi\left(w_2,q_{\mathrm{out}\,1_2}\right)\ge\Pi\left(w_2',q_{\mathrm{out}\,1_2}'\right)$.

In summary, if the optimal value corresponding to the optimal local maximum point of Problem (11), $\max\left\{\Pi\left(w_1',q_{out1_1}'\right),\Pi\left(w_2',q_{out1_2}'\right)\right\}$, exists, then $\Pi_s(\hat{w},\hat{q}_{\mathrm{out}\,1})\ge\max\left\{\Pi\left(w_1',q_{out1_1}'\right),\Pi\left(w_2',q_{out1_2}'\right)\right\}$. Combining this with the condition that $\Pi_s(\hat{w}, \hat{q}_{out1}) \geq \Pi_s\big(c_s, q^*_{out1}(c_s)\big)$, which is the optimal value corresponding to the boundary point of Problem (11), we can conclude that $\Pi_s(\hat{w}, \hat{q}_{out1})$ is the upper bound of Problem (11). Specifically, if either $(\hat{q}_{\text{out }1}, 0)$ given $\hat{q}_{\text{out }1} = q_{\text{out }1_1}$ or $\left(\hat{q}_{\text{out }1}, F_1^{-1}\left(\frac{p-c_b}{p}\right) - \hat{q}_{\text{out }1}\right)$ given $\hat{q}_{\text{out }1} = q_{\text{out }1_2}$ is optimal for the problem $\max_{q_{\text{out }1}, q_{\text{in }1} \geq 0} \Pi_b(\hat{w}, q_{\text{out }1}, q_{\text{in }1})$, then $(\hat{w}, \hat{q}_{\text{out }1}) \in \Omega_1$ or $(\hat{w}, \hat{q}_{\text{out }1}) \in \Omega_2$, so $\hat{w}$ is a feasible solution to Problem (11), and thus

$w^* = \hat{w},\ q^*_{out1}(w^*) = \hat{q}_{\text{out }1}, q^*_{\text{in }1}(w^*) = 0$ or $q^*_{\text{in }1}(w^*) = F_1^{-1}\left(\frac{p-c_b}{p}\right) - \hat{q}_{\text{out }1}$.

2) If $(\hat{w}, \hat{q}_{out1})$ does not exist, then Problem (11) has no local maxima. In this situation, there are two cases: (1) outsourcing in Stage 1 is optimal for the buyer, $w^* = c_s, \big(q^*_{out1}(c_s), q^*_{in1}(c_s)\big) = \arg\max_{q_{out1}, q_{in1} \in [0,1]} \Pi_b(c_s, q_{out1}, q_{\text{in }1})$, and (2) producing in-house in Stage 1 is optimal for the buyer; $w^* = c_s, q^*_{out1}(w^*)$ and $q^*_{in1}(w^*)$ are shown in Theorem 3.

7. Proof of Theorem 7

From above definition, for Problems (12) and (14),

$$\begin{cases} g_1(w, q_{\text{out }1}) = 0, \\ q_{\text{out }1} + \big(1 - \alpha(q_{\text{out }1})\big)F_2^{-1}\left(\dfrac{p-w}{p}\right) - (w - c_s)\dfrac{1 - \alpha(q_{out1})}{p f_2\left(\dfrac{p-w}{p}\right)} = 0, \\ J_5(w, q_{\text{out }1}) \geq 0, \end{cases}$$
$$\Rightarrow \begin{cases} G_1(w, q_{\text{out }1}) = 0, \\ G_2(w, q_{out1}) = -\dfrac{1 - \alpha(q_{out1})}{p f_2\left(\dfrac{p-w}{p}\right)} J_5(\tilde{w}, q_{\text{out }1}) \leq 0, \end{cases}$$

That is, the feasible region of Problem (12) contains the feasible region of Problem (14). Similarly, the feasible region of Problem (13) contains the feasible region of Problem (15). Therefore, if $(\tilde{w}, \tilde{q}_{out1})$ exists, then it is a feasible solution to Problem (12) or Problem (13).

From the argument in the proof of Theorem 6, we know that if either $(\tilde{q}_{\text{out }1}, 0)$ given $\tilde{q}_{\text{out }1} = q_{\text{out }1_3}$ or $\left(\tilde{q}_{\text{out }1}, F_1^{-1}\left(\frac{p-c_b}{p}\right) - \tilde{q}_{\text{out }1}\right)$ given $\tilde{q}_{\text{out }1} = q_{\text{out }1_4}$ is optimal to the problem $\max_{q_{\text{out }1}, q_{\text{in }1} \geq 0} \Pi_b(\tilde{w}, q_{\text{out }1}, q_{\text{in }1})$, then $(\tilde{w}, \tilde{q}_{\text{out }1}) \in \Omega_1$ or $(\tilde{w}, \tilde{q}_{\text{out }1}) \in \Omega_2$, where Ω_1 and Ω_2 are defined in proof of Theorem 6 . That is, $\tilde{q}_{\text{out }1} = q^*_{\text{out }1}(\tilde{w})$, so $\tilde{w}$ is a feasible solution to Problem (11) with $g_1(\tilde{w}, \tilde{q}_{out1}) = \frac{\mathrm{d}q^*_{out1}(w)}{\mathrm{d}w} = 0$, and $\Pi_s(\tilde{w}, \tilde{q}_{out1})$ is a lower bound of Problem (11).