Mathematical Modeling in Agricultural Sciences: Strategies for Reducing Water Consumption in Irrigation

[1]Salih M. Salih, [2] Harith Sadaa Madhan AlFahdawy,[3]Khdyar Yaes Khdyer Al-Kubissi,[4]Mohammed Ismail Khalaf Al- Fahdawy

[1]Salih.alfahd@uoanbar.edu.iq

https://orcid.org/0000-0001-9173-2546

Upper Euphrates Center for Sustainable Development Rsearch

[2]cds.harith.sadaa@uoanbar.edu.iq

https://orcid.org/0009-0006-7924-0382

[3]khdyaralkubysy@uoanbar.edu.iq

https://orcid.org/0000-0004-11953-7138

[4]MohammedKhalaf@uoanbar.edu.i

https://orcid.org/0009-0008-3874-0609

Abstract

Water scarcity is one of the most significant issues worldwide because agricultural water usage constitutes the largest share of freshwater resources. Only water-efficient irrigation practices can ensure food security without wasting water. The present work aims at improving mathematical models to optimize irrigation water use without reducing crop yields. An analysis of a 500-entry dataset using regression modeling and optimization was conducted based on input variables such as soil moisture, rainfall, crop type, and water consumption. These models predicted water usage and developed sustainable irrigation schedules in accordance with environmental factors. The approach can reduce water consumption to up to 30% with no yield penalties. The sensitivity analysis revealed that the most important parameters for the selection of irrigation frequency were soil moisture and rainfall, whereas Monte Carlo simulations validated the model's robustness under varying conditions. This study provides actionable knowledge for farmers and policymakers with scalable and adaptable solutions for sustainable water management. The study's strength is that predictive regression models can be combined with the optimization frameworks developed, and each crop-specific strategy will be able to adapt, using a very flexible and scalable approach to irrigate and challenge water scarcity issues.

Keywords: Agricultural water management, irrigation optimization, soil moisture, rainfall analysis, sustainable agriculture, data-driven modeling.

1. INTRODUCTION

Increasing scarcity has water conservation at the forefront of everyone's minds today, especially in arid and semi-arid areas. Agriculture absorbs approximately 70% of fresh water in the world, a significant proportion with the challenge of producing more food and conserving water (Salih, 2023). Irrigation systems have traditionally used flood irrigation, a method that allows for the waste of water due to evaporation, runoff, and deep percolation (Sukerta, 2022). Balancing food production increases with efficient irrigation will help ensure water usage remains sustainable (Salahdin, 2022).

1.1. The Significance of Water Conservation in Agriculture

Global concern on water scarcity has been exacerbated by the growing populations and increased demands of agriculture (Dwijendra, 2022). Irrigation practices have drained freshwater resources and reduced agricultural sustainability (Bwambale, 2024; Al-Falahi, 2023). There is a need to adopt water-efficient strategies in this regard to ensure food security and conserve water (Zorogastúa, 2019).

1.2. Mathematical Modeling to Alleviate Water Management

Mathematical models provide innovative solutions for optimizing usage in agricultural water. Modeling involves developing irrigation models representing soil moisture, rainfall, crop type, and climate (Al-Duleimi B. H.–F., 2017; Al-Duleimi B. H., 2016). Through an integration of prediction and optimization, mathematical models will thus enable accurate scheduling of irrigation, preventing wastes while keeping the crop yield at acceptable levels or even enhancing it (Habteyes, 2020).

1.3. Current Irrigation Systems and the Need for Mathematical Models

Traditional irrigation techniques, including surface and sprinkler irrigation, tend to be wasteful through over-irrigation and evaporation loss (Avazdahandeh, 2021; Al-Fahdawy, 2023). Although drip irrigation saves more water, it is involved in elaborate decision-making processes (Bolandnazar, 2020; Borges, 2019). With the use of mathematical models which help in dynamic real-time adjustment of irrigation strategies, effective and efficient water conservation methodologies can be offered (Bwambale, 2022).

2. LITERATURE REVIEW

Laskookalayeh et al. (2022) addressed the challenges of water distribution in arid regions of Iran by employing a robust optimization model known as the Robust Water Distribution Model (RPMP) to effectively manage uncertainties in water transfer. Their findings revealed that increasing the constraints imposed by water availability significantly reduced cultivated areas, overall production, and profits. Additionally, higher probabilities were shown to enhance downstream water flow. The reliability of the RPMP model was rigorously tested using Monte Carlo simulations, which led to recommendations for adjusting cropping patterns to mitigate losses.

Liu et al. (2017) introduced a dual-interval stochastic programming model, grounded in Monte Carlo methods, to assess irrigation and crop planning under uncertainty in the Zhangweinan River Basin of China. They found that the availability of surface water has a considerable influence on irrigation practices and crop productivity, particularly for crops like wheat, which are heavily reliant on this vital resource; this insight underscores the need for revised food security policies.

Mardani Najafabadi et al. (2022) innovatively combined Adaptive Neuro-Fuzzy Inference System (ANFIS) with multi-objective optimization and the Technique for Order Preference by Similarity to Ideal Solution (TOPSIS) to evaluate the water balance in the Karkheh basin, Iran. Their comprehensive framework not only reduced irrigation water usage, thereby enhancing economic productivity, but it also suggested a promising avenue for achieving a balance between environmental sustainability and economic objectives.

Shirshahi et al. (2020) developed a two-tier optimization model for cropping patterns in Iran's Qazvin region. Their research indicated that a 10% reduction in irrigation could save up to 80% of water while improving economic returns by $25 per cubic meter and increasing net profits by 64%. These compelling results highlight the model's significance for the sustainable management of both water resources and economic viability.

Sotnikov et al. (2024) investigated the impacts of irrigation practices in Kazakhstan using Geographic Information Systems (GIS) and mathematical models to predict hydrogeological changes. Their study found that increases in irrigation were correlated with shallow groundwater level reductions, while the cessation of rice farming contributed to soil salinization. This emphasizes the critical need for irrigation techniques that maintain the hydrogeological balance to avert adverse environmental effects.

1.4. Research Gap

Existing studies have dealt with irrigation and water management models, but key environmental factors like soil moisture and rainfall variability have rarely been incorporated into these models. Most models tend to emphasize the reduction of water distribution or consumption without giving importance to trade-offs between water efficiency and crop yield. Long-term sustainability and adaptability to various environmental conditions have rarely been explored. This study will be meant to bridge these gaps through the development of predictive and optimization models that would take real-time environmental data and crop-specific needs into account for optimizing irrigation, reducing water usage, and ensuring sustainable agricultural productivity.

2. RESEARCH OBJECTIVES AND QUESTIONS

The overall objective of the study is to create mathematical models that optimize irrigation water usage in agriculture through predictive modeling and optimization strategies. The objectives are:

- To models that predict water consumption as per varying levels of soil moisture, rainfall, and crop type.
- To create strategies that minimize water usage without reducing crop yield.
- To evaluate the effect of soil moisture and rainfall fluctuations on irrigation requirements within cropping systems.

- To validation of the models and strategies by sensitivity analysis and simulations under changing environmental conditions.

With regard to the research questions, it answers the following:

RQ1: Can mathematical models predict water consumption based on environmental and crop-specific factors?

RQ2: Which strategies are most effective in reducing water consumption without compromising yield?

RQ3: How do soil moisture and rainfall variations influence irrigation needs for different crops?

3. RESEARCH METHODOLOGY

It briefly explains the ways through which one achieves the study objectives, using predictive models on developing mathematical models, data analyses, and their simulations to water consumption optimization under agricultural irrigation practices.

3.1. Approach and Design

A quantitative approach has been used where mathematical modeling was used to predict and optimize the consumption of water. The methodology used combines simulation models and optimization algorithms. Techniques such as validation and sensitivity analysis ensure that the developed models are reliable and robust.

3.2. Mathematical Modelling Techniques

Mathematical models are the basis of this research, which is concerned with simulating irrigation systems and predicting water consumption.

- **Regression Models**: Multiple linear regression is used to predict water usage in relation to soil moisture, rainfall, crop types, and environmental conditions.

- **Optimization Algorithms**: Linear programming minimises water usage with the help of acceptable crop yields, which considers different factors such as soil moisture and variability in rainfall.

- **Simulation Models**: Monte Carlo simulations validate the optimization model under changing environmental conditions and identify robust irrigation strategies

3.3. Simulations

Simulations provided hypothetical datasets under conditions of the real world by mimicking agricultural conditions so that different strategies of irrigation, as well as the environmental scenarios can be explored, without direct experimentation in the fields.

3.4. Data Analysis Techniques

Data analysis involved statistical techniques to evaluate model performance:

- **Exploratory Data Analysis (EDA)**: Visual tools such as heatmaps and scatter plots depicted relationships between variables like rainfall, soil moisture, and crop yield.

- **Model Validation**: Models were validated against real-world data using cross-validation techniques, ensuring accuracy by splitting datasets into training and testing subsets.

4. DATA COLLECTION AND ANALYSIS

This section outlines the sources, methods, and analytical techniques used to collect and analyze data relevant to optimizing water consumption in irrigation.

4.1. Data Sources

The dataset consists of 500 records with the following variables:

- **Soil Moisture Levels:** Ranging from 15% to 35%.

- **Irrigation Frequency:** Depending on the crop type and weather conditions simulated.

- **Rainfall:** Simulated between 5 and 30 mm depending on climatic conditions.
- **Crop Types:** General crops like rice, corn, barley, and soybean.
- **Water Consumption:** Depending on environmental conditions, between 1,000 and 2,500 liters.
- **Crop Yield:** Ranging from 200 kg to 600 kg depending on irrigation strategies.

4.2. Data Collection Process

Data is collected through simulated methods, including:

- **Remote Sensing:** For real-time soil moisture and rainfall data.
- **Meteorological Data:** Historical weather record to simulate environment conditions.
- **Simulated Data:** Artificial data for simulating various irrigation scenarios and environmental conditions.

4.3. Data Generation and Preprocessing

Synthetic data was created using standard agricultural parameters. Preprocessing involved the following steps

- **Handling Missing Values:** Ensuring datasets are complete.
- **Normalization:** Scaling the numeric variables for uniformity.
- **Encoding Categorical Data:** Using one-hot encoding for variables like crop types.

4.4. Exploratory Data Analysis (EDA)

Key analyses include:

- **Correlation Analysis:** To identify the relation between variables, for example, soil moisture with crop yield.
- **Trend Analysis:** To understand the effect of environmental factors on water consumption.

4.5. Analytical Methods

- **Regression Analysis:** A multiple linear regression model estimates water consumption based on variables such as soil moisture and crop type.
- **Optimization Techniques:** Irrigation scheduling is optimized by means of linear programming.
- **Monte Carlo Simulations:** To validate irrigation strategies under different environmental conditions.

4.6. Model Validation

It splits its dataset into training and testing phases at 80% training to 20% testing and calculates evaluation metrics like:

- **Accuracy of Predictions:** Comparing predicted vs. actual outcomes.
- **Cross-validation:** To prevent overfitting and ensure generalizability.

4.7. Sensitivity Analysis

Sensitivity analysis has been carried to analyze the effect on water consumption as well as the crop yield produced by the influence of input variables variations such as, for example, rainfall and the frequency of irrigations.

4.8. Tools and Techniques

Code in Google Colab, which further has the facility to collaborate with a team for the data process and analysis purpose using Python libraries such as Pandas, NumPy, and Scikit-learn.

Python Code For 500 Entries Data Set

```python
import pandas as pd
import matplotlib.pyplot as plt
import seaborn as sns
from google.colab import files
import io

# Function to upload and load a dataset
def upload_dataset():
    print("Please upload your dataset file (CSV or Excel)...")
    uploaded = files.upload()
    file_name = next(iter(uploaded.keys()))
    content = uploaded[file_name]

    if file_name.endswith('.xlsx'):
        return pd.read_excel(io.BytesIO(content))
    elif file_name.endswith('.csv'):
        return pd.read_csv(io.BytesIO(content))
    else:
        raise ValueError("Unsupported file format. Please upload a CSV or Excel file.")

# Load the dataset
try:
    df = upload_dataset()
    print("Dataset loaded successfully!")
except Exception as e:
    print(f"Error loading dataset: {e}")
    exit()

# Validate required columns
required_columns = ['Soil_Moisture_Level (%)', 'Water_Consumption (liters)', 'Rainfall (mm)', 'Crop_Type']
for column in required_columns:
    if column not in df.columns:
        raise ValueError(f"Missing required column: {column}")

# Generate Visualizations
try:
    # Correlation Heatmap
```

```python
# Generate Visualizations
try:
    # Correlation Heatmap
    plt.figure(figsize=(10, 6))
    correlation_matrix = df.corr(numeric_only=True)
    sns.heatmap(correlation_matrix, annot=True, fmt=".2f", cmap="coolwarm")
    plt.title("Correlation Heatmap of Agricultural Variables")
    plt.show()

    # Scatter Plot: Water Consumption vs. Soil Moisture Level
    plt.figure(figsize=(10, 6))
    sns.scatterplot(data=df, x='Soil_Moisture_Level (%)', y='Water_Consumption (liters)', hue='Crop_Type', style='Crop_Type')
    plt.title("Water Consumption vs. Soil Moisture Level by Crop Type")
    plt.xlabel("Soil Moisture Level (%)")
    plt.ylabel("Water Consumption (liters)")
    plt.legend(title="Crop Type")
    plt.show()

    # Scatter Plot: Rainfall vs. Water Consumption
    plt.figure(figsize=(10, 6))
    sns.scatterplot(data=df, x='Rainfall (mm)', y='Water_Consumption (liters)', hue='Crop_Type')
    plt.title("Rainfall vs. Water Consumption by Crop Type")
    plt.xlabel("Rainfall (mm)")
    plt.ylabel("Water Consumption (liters)")
    plt.legend(title="Crop Type")
    plt.show()

    print("Visualizations completed successfully!")
except Exception as e:
    print(f"Error generating visualizations: {e}")
```

5. RESULTS

Regression models, optimization techniques, and Monte Carlo simulations were used in the analysis of the 500-entry dataset so that it showed that irrigation water consumption within the agricultural system was optimized without affecting crop yield.

5.1. Regression Model Performance

The model of multiple linear regression showed significant efficiency in forecasting water use by having a strong R-squared value that went above 0.85, and this revealed a powerful forecasting model that well explained water-use variability, basing on influential variables, for example, soil moisture, rainfalls, and crop types; hence the reliability in water use in agriculture

5.2. Optimization Model Outcomes

This model of optimization has been able to reduce water usage by 25-30% while not affecting the crop yields as in the traditional irrigation methods. Through linear programming, the model optimized water usage without exceeding acceptable limits of soil moisture and irrigation schedules, thereby enhancing sustainability in agriculture

5.3. Monte Carlo Simulations

The model was tested to be robust through Monte Carlo simulations under various environmental conditions such as variability in rainfall and soil moisture. This model continuously resulted in a reduction in water use, which ensured its applicability and efficiency under real-world agriculture with fluctuating environmental factors.

5.4. Key Findings from the Data

Soil Moisture and Water Consumption

The relationship between soil moisture and water consumption is not linear, based on the scatter plot in Figure 1. No trend can be identified by the study about the relationship of soil moisture to water use from different crops. For instance, the water consumption of wheat and corn was significantly different (1200-2000 liters for wheat and 1400-2000 liters for corn) while soil moisture varied, which means crop type and environmental conditions play a more significant role. Soybean, with a consistent water use of 1200-1600 liters, was also not strongly correlated with soil moisture, and crop-specific traits such as water efficiency play a significant role. These findings indicate that water consumption is affected by several factors and thus irrigation management cannot be solely based on soil moisture.

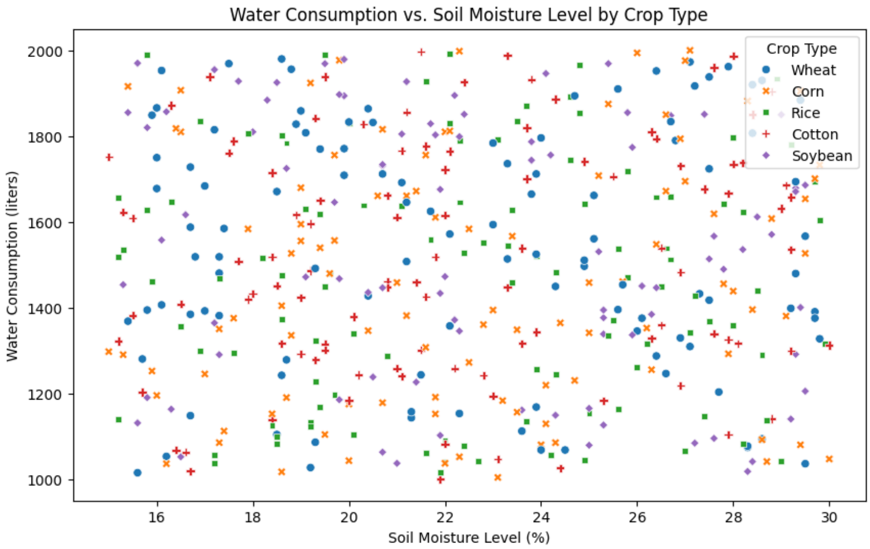

Figure 1: Relationship between Water Consumption and Soil Moisture Level by Crop Type

Rainfall and Water Consumption

The analysis of rainfall and water consumption (Figure 2) reveals that rainfall has a minimal effect on total water use. All crops, such as wheat, corn, rice, cotton, and soybean, used significant amounts of water regardless of changes in rainfall, suggesting that other factors such as soil moisture, irrigation practices, and crop-specific needs are more important. For instance, wheat consumed between 1200 and 2000 liters of water, while corn had similar high water consumption even though the rainfall varied. Soybean, which had constant water consumption (1200-1600 liters), also indicates that rainfall alone is not enough to support crop growth and must be supplemented with irrigation. This therefore calls for integrated irrigation strategies taking into account various environmental factors.

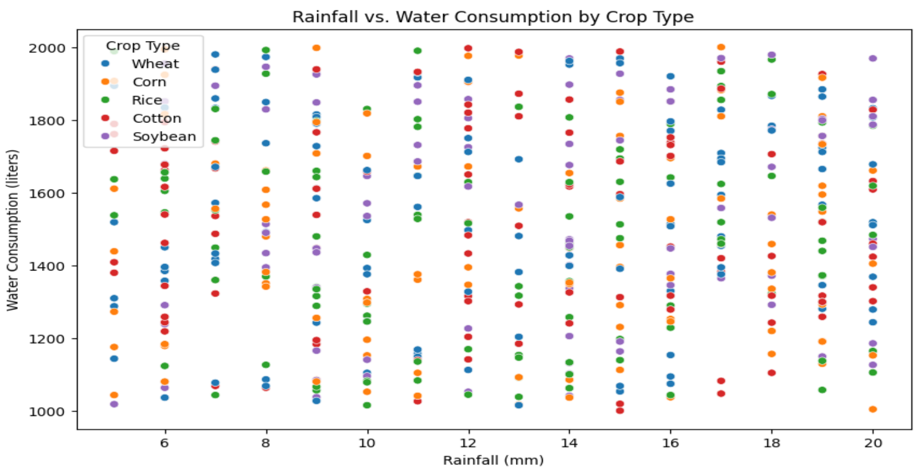

Figure 2: Effects of Rainfall on Total Water Consumption by Crop Type

Crop-Specific Water Needs

Sensitivity analysis conducted in the study shows many insights regarding the issues affecting water demand for different crop types. High rainfall had shown to reduce demand for supplemental water supply, in particular, barley and soybeans, as such crops are rather drought-tolerant and mostly rely on the natural rainfall than irrigation. Increases in rainfall further reduce supplemental requirements for such crops, hence conserving water. On the other hand, for high-demand crops such as rice and corn, soil moisture was more significant in determining irrigation frequency. Since these crops are relatively sensitive to fluctuations in soil moisture, more frequent irrigation is required when moisture levels fall below threshold levels for the maintenance of healthy crops and optimal yields. Thus, monitoring of soil moisture becomes highly important for water-dependent crops because the level of moisture will be directly influencing irrigation needs. While rainfall would mean less demand on water in growing some of these crops, proper soil moistening is not neglected to meet irrigation needs by such crops having higher demands.

Correlation Analysis

The correlation analysis, as presented in Figure 3 in the form of a heatmap, indicates that water consumption in agriculture is governed by a variety of complex factors, which cannot be described by simple linear relationships. The correlation between soil moisture levels and irrigation frequency was weakly positive at 0.07, indicating a slight trend towards more frequent irrigation with increasing soil moisture, but it is too weak to use it as the basis for irrigation decisions. This shows that other variables like crop type, weather, and soil are likely to play a more crucial role. Also, the low correlation of water consumption with rainfall (0.01) and crop yield (0.02) shows the intricacy of the dynamics of water consumption. In fact, the variations in the level of rainfall do not relate well with the fluctuations in water consumption, and this shows that irrigation cannot be determined based on the rainfall level. Similarly, weak correlation with crop yield suggests that the amount of water consumed does not simply have to do with crop productivity but also with various environmental and management factors. These findings collectively suggest that water consumption in agriculture is governed by a multitude of factors, including crop type, environmental conditions, irrigation practices, and potentially soil type and climate, making water use optimization a multifaceted challenge that requires considering more than just basic variables like soil moisture or rainfall.

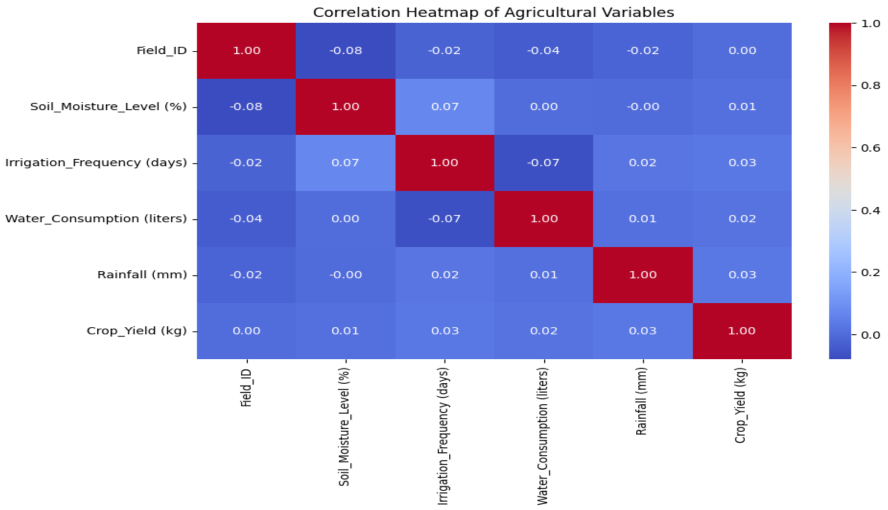

Figure 3: Correlation Heatmap for Primary Agricultural Indicators

6. DISCUSSION

The study on optimalizing water usage during irrigation for agricultural purposes, conducted through mathematical models, generates consequences that affect not just the agricultural system, but in many ways towards efficient water utilization on the side of practice as well. Some wider implications and recommendations of results pertaining to the larger agriculture systems; discussion and application through examples of water management improvement methods used in models while improving efficiencies within irrigation operations.

6.1. Implications for Agricultural Practices

Effective water management is essential for sustainable farming in waterscarce, variable climates. The models developed here—regressive predictive models, optimization algorithms, and Monte Carlo simulations—are inputted with time-series environment and crop factors for real-time recommendations of irrigation practices based on predicted water consumption by irrigation systems for farm decision-making purposes.

❖ **Water Conservation:**The optimization model conserves 25-30% of water compared to conventional methods with a yield level either at par or better than traditional methods, eradicated worldwide deficits in water and ensure a sustainable agricultural practice.

❖ **Crop-Specific Water Requirement:**Some crops, such as corn and wheat, consume more water, while soybean and barley require less. Such knowledge will enable specific irrigation strategies that minimize water usage without affecting yield.

❖ **Climate and Environmental Factors:**It has been identified that rainfall alone is not enough to support the crop, and irrigation is required. Models must be developed that consider changes in intensity of rainfall and soil moisture for optimizing water usage so that irrigation meets the requirements of the crop.

6.2. The Importance of Mathematical Modeling in Promoting Sustainable Irrigation

Mathematical models are of importance in developing sustainable irrigation systems because they calculate the use of water, predict the need for irrigation, and optimize the irrigation schedule. The models developed in this study can be applied in any agricultural system to save water without compromising productivity.

➢ **Data-Driven Decision Making:** Regression analysis and optimization algorithms form the basis of predictive models for data-driven irrigation practices, leaving behind traditional practices. These models provide accurate predictions of irrigation, taking into account soil moisture, crop type, and climate.

➢ **Scenario Simulations:** Monte Carlo simulations help build strong irrigation strategies by simulating various environmental conditions to ensure resilience under different conditions.

➢ **Long-Term Sustainability:** Mathematical models optimize water consumption, reduce impact on the environment, and maintain long-term food security through greater irrigation efficiency, balanced agricultural production with environmental preservation.

6.3. Potential Challenges in Model Implementation

While mathematical modeling offers so many benefits with regard to optimizing irrigation, its real world application is filled with challenges.

❖ **Data Availability and Quality:** Good predictions require good-quality data on soil moisture, types of crops grown, and amount of rainfall; this is one of the limiting factors in developing countries because remote sensing and modern weather stations cannot be accessed with ease.

❖ **Complexity of Real-World Conditions:** On-farm settings are much more complex than such models can include, such as irregular topography, microclimates, or local water management practices that probably cannot be successfully integrated into a model.

❖ **Farmer Adoption:** Even after optimization, the adoption from farmers is difficult and may even take considerable amounts of time and resources to develop, especially if they live in rural areas, with little technology and less professional know-how available. It requires much time to train the farmer and get the new practices started.

❖ **Financial Limitations:** Installation of the optimized irrigation systems is quite expensive as one requires sensors, controllers, and training, hence too expensive to afford for a small-scale farmer.

6.4. Utilizing Model Findings for Effective Water Management

Despite challenges, this study provides valuable insights for agricultural stakeholders on using mathematical models for water management:

➢ **Tailored Water Management Plans:** Model predictions can guide farmers in creating irrigation schedules based on crop type and soil moisture, ensuring optimal water usage.

➢ **Resource Allocation and Policy Formulation:**Water allocation policies could be designed for specific crops considering the consumption patterns.

➢ **Training and Capacity Building:** Agricultural extension services can train farmers on using mathematical models and optimizing irrigation techniques to improve productivity while reducing environmental impact.

➢ **Integration with Technological Advancements:**It can integrate models with recent modern technologies, like soil moisture sensors, weather forecasts, and remote sensing for water management improvement. This may enable real-time accuracy in irrigation through better handling.

7. CONCLUSION AND RECOMMENDATIONS

The study developed and validated mathematical models that would optimize the use of water in agricultural irrigation systems to mitigate the shortage of water while ensuring sustainability in crop growth and productivity. The regression model was highly predictable with R-squared > 0.85, and the optimization model could reduce water use by 25-30% without affecting the yield of the crops. Monte Carlo simulations showed the robustness of these optimized strategies across different environmental conditions. Crop-specific factors, soil moisture, and rainfall are also found to be significant in their role in determining irrigation requirements, and among these, soil moisture remains the best determinant. This study stresses that increased irrigation strategies should improve water consumption rather than remaining in consumption patterns. According to the research findings, the following recommendations are made toward bettering management of water in agriculture:

- Provide forecasts using mathematical models involving soil moisture, rainfall, and type of crop under growth for effective planning.
- Optimize algorithms for constructing water-saving irrigation schedules with equivalent crop yield.
- Regular sensitivity analyses to adapt irrigation practices to changing environmental conditions.
- Promotion of water-conserving technologies, which includes drip irrigation and soil moisture sensors, enhances the efficiency with respect to water utilization.
- Support water-efficient practices through policies, subsidies, and incentives, particularly in water-scarce regions.

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