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A Novel Computational Model of Insulin-Glucose Chaotic System to representβ-Cells Effects, Type 1 Diabetes Mellitus (T1DM) and Hyperglycemia

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***Abstract*—** *The math model is a very good way to look at how glucose and insulin work together. This work brings in a new math model of control of insulin and glucose in the human body, based on the well-known Prey-Predator model. Chaos is one of the known things in bio systems and hard systems from the info that is out there. The results of this study match up similarly with the known results, showing that the insulin-glucose control system is a very hard one in different backgrounds. This study, then, might make an extra part of seeing how reg system work in diabetes; mainly Type 1 diabetes and hyperinsulinemia.*

Keywords: *Biological modeling, Chaotic systems, Chaos, Diabetes, nonlinearity,complexity.*

I. INTRODUCTION

Blood glucose is the most vital energy provider for the cells of neurons within the brain and other body cells. Simple sugars or carbohydrates are known as sources of life and energy for those cells. The two endocrine hormones that maintain glucose at relatively constant levels are synthesized by the pancreas, glucagon (excitatory) and insulin (inhibitory). The hormones are synthesized by alpha cells and beta cells, respectively. Lethal blood glucose oscillations manifest two distinguishable pathophysiologies: hypoglycemia and Type 1 Diabetes Mellitus (T1DM). It is the disease that belongs to the most devastating disease group. By the year 2016, more than twenty-two million Americans had been diagnosed with diabetes. Computational models are tools increasingly appreciated for their usefulness in representing, predicting, and gaining control over real, highly complex systems where experimentation is infeasible. For biological matters, one can study the effects of many drugs without setting living or viable specimens under experimental conditions, and the unnecessary side effects that may be probable. Ackerman et al. [4] proposed a linear differential system of two equations for the glucose and insulin concentrations in the body. In their model formulation, the authors placed the main emphasis on the physiological interpretation of the data obtained in the study and the parameters included. The three-dimensional systems of Bajaj et al. proposed the β-cell effect on insulin secretion in this approach, which uses a nonlinear model. Results of that work were compared with the experiments made earlier in healthy and protein malnourished people. In this work, some investigators have contemplated the formulation of Delay Differential Equations as a means to represent the time lag associated with the production of insulin. This paper will introduce a new concept aimed at clarifying the detailed mechanisms involved in the control of glucose, insulin, and β-cells in the control system. A new messy model for the moves of insulin and glucose has been presented in Section II. The present work has undertaken an analysis of the statistical and dynamical characteristics of the system through an examination of its stability, bifurcation diagram studies, and Lyapunov exponent calculations; these assessments were performed on physiologically significant parts of the system. In Section V, the attractor is transformed into three dimensions in visually attainable 3D form with the use of the RD-CNN method The obtained results are discussed in Section VI.

II. NEW MATHEMATICAL CHAOTIC MODEL FOR INSULIN-GLUCOSE REGULATORYSYSTEM

In order to present a novel framework, we examine the dynamic interplay among concentrations of insulin, glucose, and β-cells. In this particular context, the selection of physiologically significant factors has been taken into account in order to propose three-dimensional differential equations. It is important to acknowledge that the anticipated

model will be required to demonstrate behavioral reactions of the insulin-glucose regulating system. In order to ensure meaningful replies, it is imperative to establish appropriate values for the parameters. As an illustration, each of the three variables inside this system represents the concentrations of certain substances within the human body. The computational model developed for the insulin-glucose regulation system is outlined as follows.

$$\dot{X} = -R_1X(t) + R_2Y(t) + R_3Y(t)^2 + R_4Y(t)^3 + R_5Z(t) + R_6Z(t)^2 + R_7Z(t)^3 + R_{18}$$

$$\dot{Y} = -R_8X(t) - R_9X^2(t) - R_{10}X^3(t) - R_{11}Z(t) - R_{12}Z^2(t) - R_{13}Y(1-Y) + 0.5^{\exp(Y)} + R_{19}$$

$$\dot{Z} = R_{14}Y(t) + R_{15}Y^2(t) + R_{16}Y^3(t) - R_{17}Z(t) \ldots..(1)$$

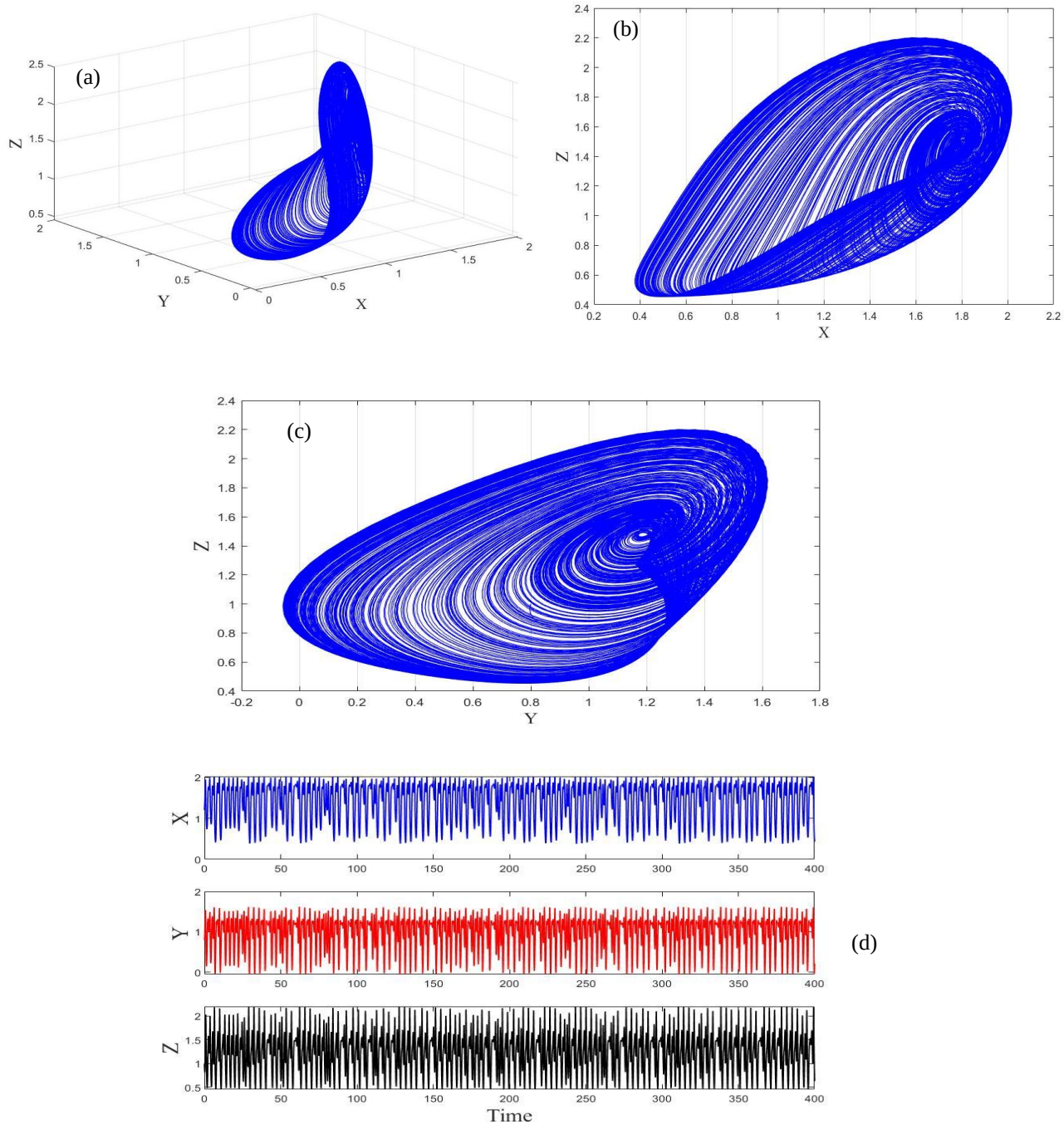

Figure 1. (a) 3D view of the chaotic response of theglucose- insulin regulatory. (b)Chaotic response in $X - Z$ space and (c)Chaotic response in$Y - Z$. (d) Positive time series.

This model represents the concentration of insulin, the population density of β-cells, etc. It is assumed that R1 represents the 'decreasing rate of change of insulin concentration' and is a function depending on the level of insulin concentration at the time. Augmentation of insulin concentration with that of glucose concentration is given by R2, R3, and R4. Augment of β-cell concentration with that of insulin concentration is given by R5, R6, and R7. R8, R9, and R10 relate to the rate of decrease in glucose concentration due to increasing levels of insulin. The increase in glucose concentration due to the presence of β-cells is modeled by R11, R12, and R13. Specific rates of increase in the proliferation of β-cells due to higher levels of glucose are described by R14, R15, and R16. Rate of the decreasing β-cells at their current level is what is indicated by R17. β-cells reduction is what the variable R18 stands for, while R19 shows the rate of glucose going up in no presence of insulin and β-cells. A figure depicts the attractor response of the system.

A. Stability Analysis of the proposed system

In order to analyze the dynamic system, focus should be centered on Table 1. The dynamics of the chaotic system can be examined by controlling the eigenvalues of the corresponding Jacobian matrix at each equilibrium point. The existence of positive fixed points in time series is important due to the biological and organic significance of the variables involved. The system has a unique positive equilibrium point expressed asas E^*=(I^*, G^*,z^*)=(1.800,1.190,1.49). The Jacobian matrix of the system described in equation (2) produces eigenvalues λ_1=4.67+16.04i, λ_2=4.76-16.04i, and λ_3=-12.38. The results of the stability analysis suggest that the equilibrium in question is stable.

$$J = \begin{pmatrix} -R_1 & R_2 + 2R_3Y + 3R_4Y^2 & R_5 + 2R_6z + 3R_6z^2 \\ -R_8 - 2R_9X - 3R_{10}X^2 & (-R_{13}(1-2Y) - 0.67(0.5^{\exp(Y)})(exp^Y)) & -R_{10} - 2R_{11}z - 3R_{12}z^2 \\ 0 & R_{14} + 2R_{15}Y + 3R_{16}Y^2 & -R_{17} \end{pmatrix} \quad \text{.....(2)}$$

B. Bifurcations and Lyapunov Exponents Diagrams

This section presents a typical output of the bifurcation and Lyapunov exponent diagrams for R16. We shall try to relate the biological meaning of these parameters as in Figure 2. As has been discussed in earlier works, when a system shows chaotic dynamics, it is indicative of the presence of maximum entropy [10]. The study finds that chaotic dynamics are established in the system under consideration. The ailment that most people refer to sadly as a biological disease is probably depicted by Figure 1 as resulting from the breakdown of insulin in the biology. What is being hinted at is that it shows connections between bifurcation and Lyapunov exponents. The potential of Lyapunov exponents spectra is a methodology that deals with the nonlinear systems on the basis of their dynamics. The study shall be on how fast different trajectories within a chaotic system diverge and converge in space. A positive Lyapunov exponent unequivocally declares the system as truly chaotic . The phenomenon of the chaotic attractor and the tendency of the state variables show specifically associated with this collection of parameters . From this, we can thus jump to the conclusion that even slight deviations in model parameters may result in undesirable system behavior.

Table1: Parameters of the proposed system

Parameter	Value	Parameter	Value
R_1	0.30	R_{11}	1.95
R_2	0.69	R_{12}	0.75
R_3	0.27	R_{13}	-0.02
R_4	0.92	R_{14}	0.83
R_5	0.98	R_{15}	1.01
R_6	-0.62	R_{16}	1.25
R_7	-0.10	R_{17}	1.43
R_8	1.24	R_{18}	-0.83
R_9	-1.06	R_{19}	1.93
R_{10}	-0.29		

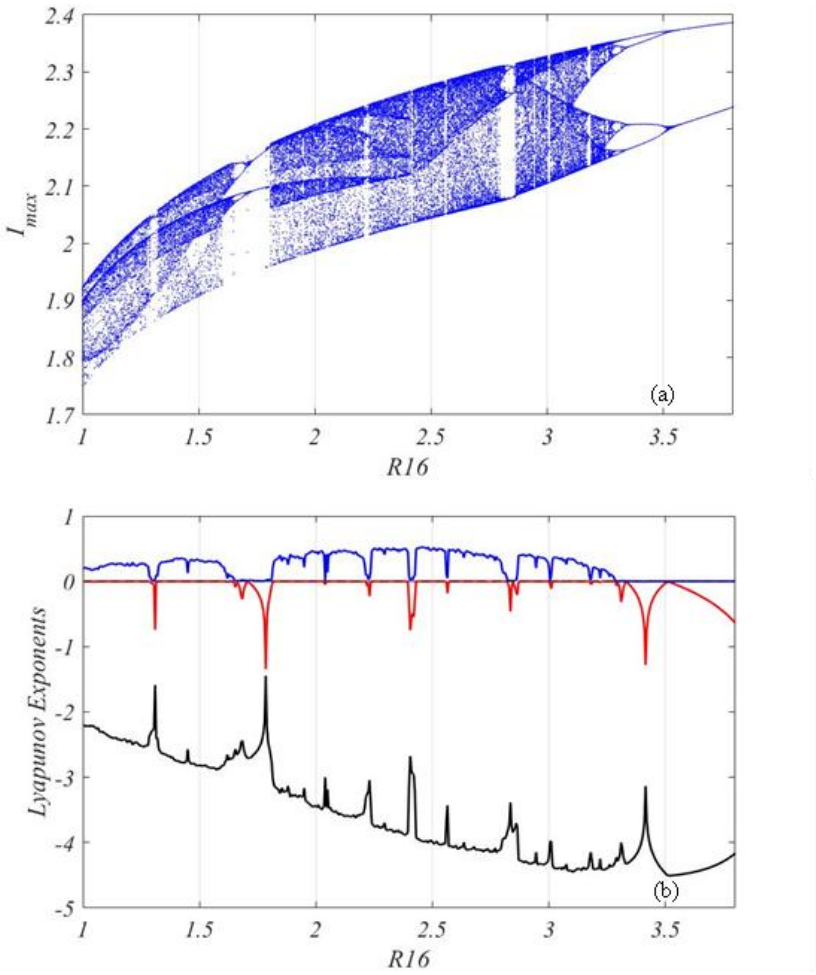

Figure 2 (a) Bifurcation diagram of the model with I_{max} when the parameter R_{16} changes and (b) Lyapunov diagram with respect to R_{16}. This parameter in $\dot{Z}$ eq. shows the increase rate of glucose concentration that may cause biological disorder.

III. RESULTS AND DISCUSSION

This section looks at the dynamic results of the model suggested. To bring in many forms of DM and other diseases into this system, we proceed to construct the bifurcation diagram of the system by varying factors of physiological importance. The bifurcation diagram gives a picture of the system analytically in how it responds to a change in the control parameter; that is, as it is increased or decreased. Changed scope and type of system response show sensitivity of the system to the given control parameter. [12].

A- Type 1 DM

This disease shows that type 1 diabetes involves an immune-mediated response that reduces the concentration of β-cells in the body. As the concentration of these cells decreases, insulin secretion is also reduced. In this respect, the model of the rate equation which now steers the changing rate of β-cells denoted by z˙ should include parameter R _8 to account for a slow fall in β-cell concentration. A decrease in this parameter will allow the system to express the effect of such a fall on the dynamic changes of glucose, insulin, and β-cells in a type 1 diabetic patient.

The bifurcation diagram, illustrating the variation of insulin concentration with the variation of parameter R_8, is presented in Figure 3. From this figure, it is clear that at very low levels of decrease in β-cells, there is a chaotic attractor of insulin concentration. Our working hypothesis would be the following: the observed chaotic insulin dynamics in day profiles are to be placed within the framework of healthy physiological variations. Chaotic dynamics of insulin disappear with β-cell loss. Dynamics of insulin are organized by a period-halving bifurcation.

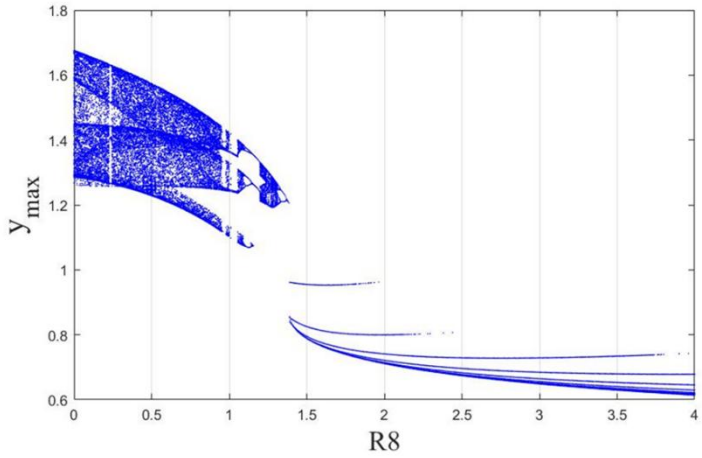

Figure 3 Bifurcation diagram of the system as the parameter R_8changes.

B- β-Cells Effect

In the onset of type 2 diabetes mellitus, there is a presence of high levels of insulin found in the blood which is called hyperinsulinemia. Although this overproduction of insulin is also found in other diseases such as hypertension and obesity, cancer, and Alzheimer's disease, it will be included in this model as parameter R4 to define that certain condition is available inside the insulin-glucose regulatory system. The bifurcation of the system is shown in Figure 4 for the values between 0 and 4.0 for R12 and R17. Chaotic behavior of the system is observed for low as well as high values of the control parameter. This is a transitional phenomenon to be considered between two discrete states.. [12].

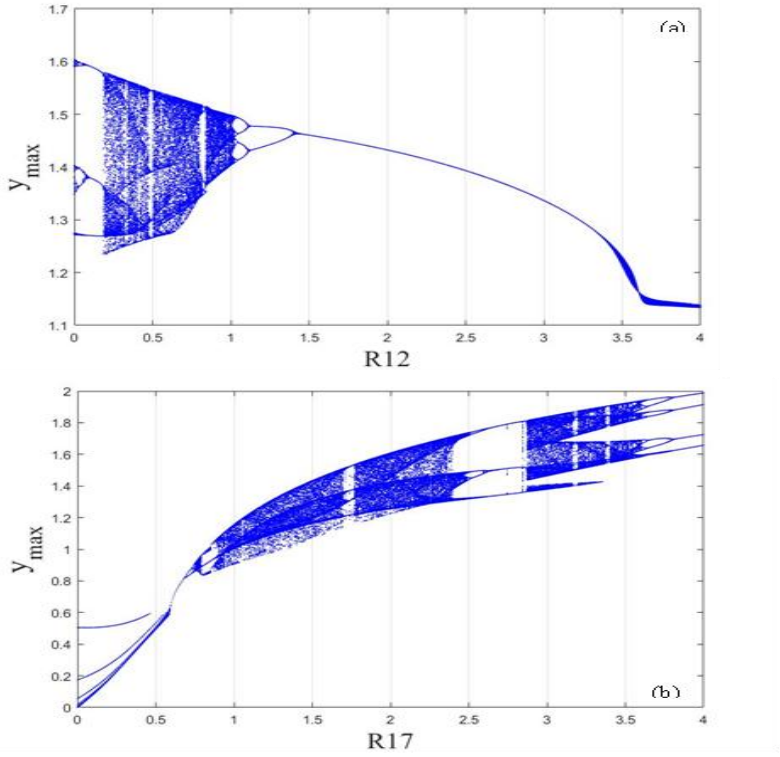

Figure 4 Bifurcation diagram of the system as the parameters R12 and R17 change. (a) This parameter (R12) shows the rate at which the glucose concentration increases when there is a decrease in the level of β-cells with respect to the Y dimension. (b) This parameter (R17) shows the rate at which the insulin concentration decreases when there is a decrease in the level of β-cells with respect to the Y dimension. (c) This parameter (R12) shows the rate at which the glucose concentration increases when the level of β-cells decreases with respect to the X dimension. (d) This parameter (R17) shows the rate at which the insulin concentration decreases when the level of β-cells decreases

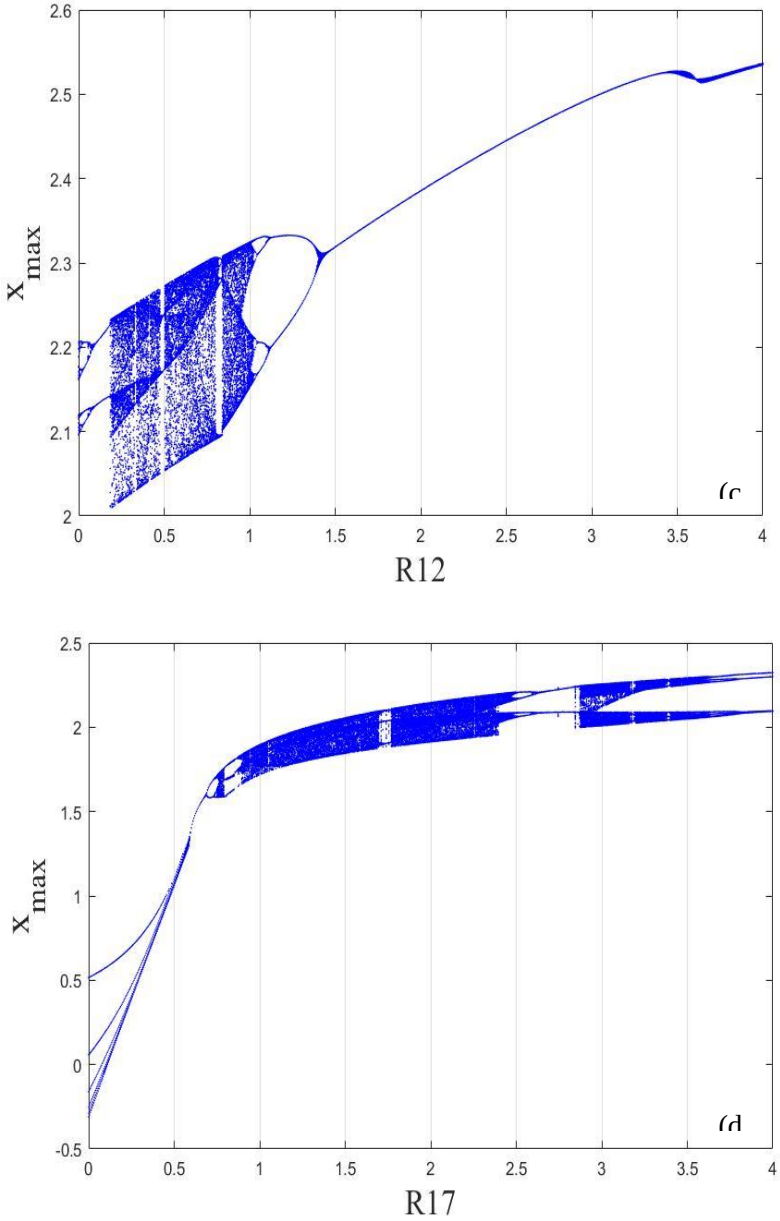

Figure 4 (Continued)

C- Hypoglycemia

People take insulin and have kidney failure, some types of cancers, or liver disease, or are a substance user such as alcohol, typically see a drop in their glucose level in the blood [17]. This is known as low blood sugar or hypoglycemia. In order to simulate this life-threatening condition, we define the variable R7 to be the rate of hypoglycemia developing due to the rise in the concentration of insulin.

As shown in Figure 5, it can be observed that with the increase in the value of the control parameter, R14, there is a decrease in the level of glucose. A decrease in the level of glucose indicates to us that the dynamics of the system are changing over from chaos to a state of periodic organized behavior. As we have mentioned earlier, the chaotic and random responses observed at lower values of the parameter can simply be considered as common occurrences that raise this value where order sets in, disappears, and there is minimal response...

[12].

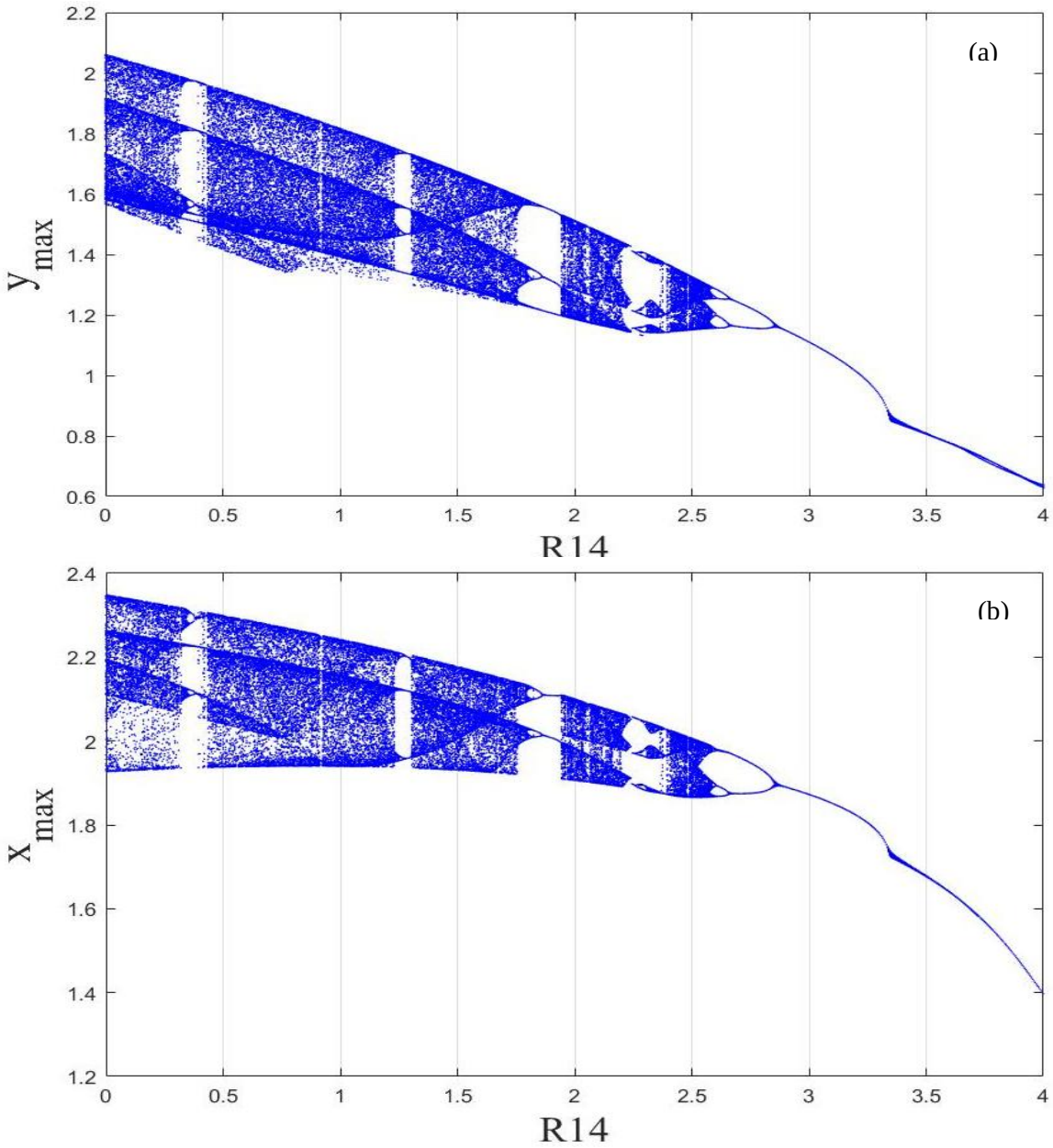

Figure 5 Bifurcation diagram of the system as the parameter R_{14} changes. This parameter (R_{14}) shows the increasing rate of β-Cells in response to reduction of the glucose level with respect to both X and Y dimensions as show in (a) and (b), respectively.

IV. Basin Of Attraction

Attractors in dynamical systems will always belong to one of these two classes. First is an attractor that can be referred to as the self-excited type of motion. An attractor is termed self-excited if its basin of attraction contains an equilibrium point. If there is no such feature of the attractor, then it is termed hidden. It can also be considered as an attractor having its basin of attraction coincident with any arbitrary open set that includes an unstable fixed point; that is a concealed attractor. As described in [19], the basin of attraction of a hidden attractor does not topologically relate with any unstable fixed point. This is what is depicted in Fig. 6. The two basins of attraction of the proposed system are illustrated in Fig. 6. The plot of the behavior of the system on the plane defined by xy at a fixed value of z, specifically z = 0.44, is given in Fig. 6. 6(a) slightly different parameter values from those listed in Table 1 being used in cases where a steady point of reference is not there notice the existence of messy pull towards goes. Figure 6 (b) what shows the boundless area in red the regular (limit cycle) in green and the equasiperiodic (tours) in black parameters used are the same as those listed in Table 1 and the value of z is

fixed at 1.04. The projection shown is in the xy-plane..

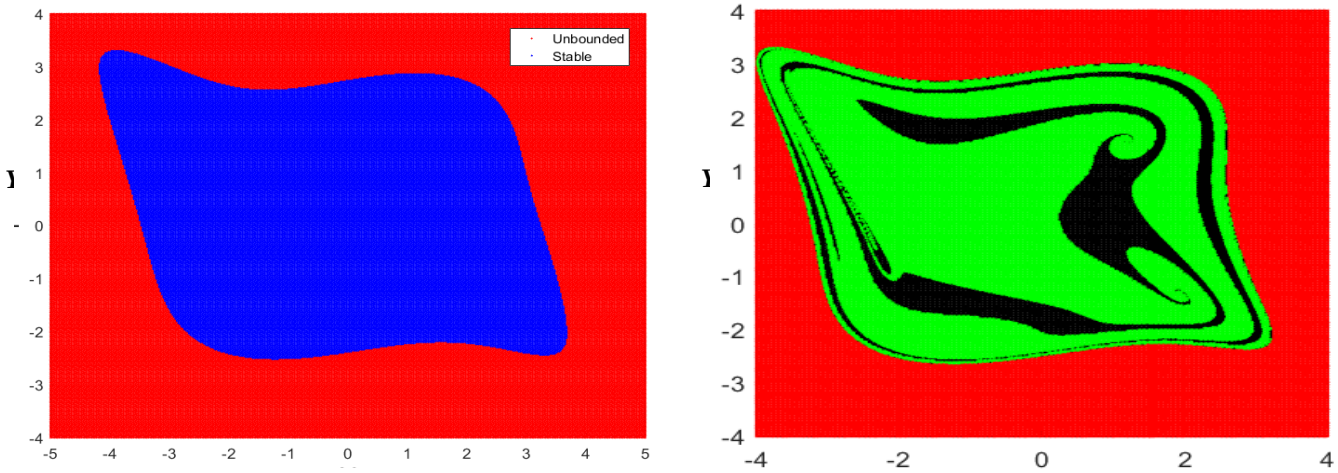

Figure 6 (a)This figure shows the basin of attraction of the system in z = 0.44, it shows that there isn't any hidden chaotic attractors with another parameters and (b) Basin of attraction of the system with parameters as in Table1 in this condition, chaotic (black), periodic (green) and unbounded (red) attractors are seen.

(a) (b)

VI. Conclusions

Diabetes mellitus is a common disease. It results from the departure of insulin and glucose levels in the blood from their healthy norm. This work introduces a new mathematical model for the simulation of diabetes alongside two other related metabolic diseases. The purpose is to qualitatively and quantitatively test the effects of some physiological factors in this model under computer simulation based on differential equations. An analysis of the bifurcation diagram of the system is, therefore, carried out with reference to the R2 parameter to be able to throw some light on the dynamic changes that are happening in type 1 diabetes. This paper presents how a healthy system can transform into a chaotic one, hence indicating dynamical sickness. To improve further analysis, one can take into consideration the effectiveness of newly proposed drugs by checking the relevant parameters inside this system since it is controllable..

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