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Exploration and Implementation of the "Problem-Oriented + Progressive Case" Teaching Model Reform: A Case Study of Probability Theory and Mathematical Statistics

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Abstract:

Mathematics is the most brilliant pearl of human wisdom. As a critical branch of modern mathematics, probability theory and mathematical statistics serve as foundational courses for engineering, economics, and management disciplines. Presently, the inherently mathematical nature of these courses often leads students to focus excessively on calculations while neglecting conceptual understanding, resulting in weak knowledge transfer abilities. Bridging "concrete phenomena" with "the principles of probability and statistics" in modeling requires high levels of various student competencies, and the disconnect between traditional teaching content and the current environment has led to outdated teaching cases. Additionally, a singular assessment method discourages student enthusiasm and results in low learning efficiency. To address these issues, our team adopted a "problem-oriented + hierarchical planning" teaching model. We restructured the teaching content, reshaped the teaching process, rebuilt online teaching resources, re-established multiple assessment methods, and integrated ideological and political elements. This reform achieved initial success and received high recognition from students. Furthermore, our team mentors guide students to participate in competitions such as the National College Students Mathematical Modeling Competition, the "Internet +" College Students Innovation and Entrepreneurship Competition, and the National College Students Mathematics Competition, thereby fostering the development of application-oriented talents.

Keywords: probability theory and mathematical statistics; Problem-oriented; Layered case

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1 Introduction

The course of Probability Theory and Mathematical Statistics is a crucial foundational mathematics course for sophomore undergraduates majoring in economics and management, engineering, and science and technology. As a key branch of modern mathematics, it primarily investigates the statistical patterns of numerous random phenomena occurring in nature, human society, and technical production processes. This discipline is extensively applied across natural sciences, social sciences, engineering technologies, and finance, while continuously integrating with other fields. Consequently, this course serves as an indispensable foundation for further studies and research in related disciplines. It plays an irreplaceable role in cultivating students' mathematical thinking and literacy, rational spirit, logical reasoning skills, abstract thinking abilities, responsiveness

to random events, and data processing capabilities. Moreover, it provides a scientific platform for training high-quality, practically skilled, and innovative talents across all majors.

2. An Analysis of Curriculum Characteristics and Learning Contexts

2.1 Curriculum Characteristics

Probability Theory and Mathematical Statistics courses possess the following characteristics regarding the nature of the course content:

Extensive use of concept formulas and high mathematical foundation requirements: The "Probability Theory and Mathematical Statistics" course encompasses a substantial number of concepts and formulas, many of which are defined or derived through rigorous mathematical methods. For instance, calculating the probability of continuous random variables within a specific range requires the application of definite integrals using probability density functions. Additionally, the definition of mathematical expectations involves knowledge of infinite series. Consequently, a solid mathematical foundation is essential for students to excel in this course. Furthermore, there is a strong correlation between chapters, such as the relationship between the total probability formula and Bayes' theorem, as well as between distribution functions and probability density functions. This interdependence necessitates that students possess robust logical thinking skills.

A broad spectrum of practical applications and stringent ability requirements: In contemporary society, with the advent of the big data era, the principles and methodologies of probability theory and mathematical statistics have found extensive application across diverse sectors including finance, insurance, economics, enterprise management, industrial and agricultural production, medicine, geology, meteorology, natural disaster prediction, and artificial intelligence. Students are expected to abstract a wide array of practical problems into mathematical models grounded in probability theory and mathematical statistics, subsequently leveraging their acquired knowledge to analyze and resolve these issues. This process demands a high level of proficiency in abstract problem formulation, mathematical modeling, data collection, analysis, and processing.

2.2Analysis of the Learning Situation

The participants in this course are sophomore students majoring in economics and management, as well as engineering and science. Based on observations and analyses by the course team over recent years, these students exhibit several notable characteristics:

A relatively weak foundation in mathematics, insufficient abstract and logical reasoning skills, and a significant fear of challenging material. According to the statistics of higher mathematics examination in our school, the failure rate of higher mathematics examination is as high as more than 40%, indicating that many students struggle with fundamental higher mathematics concepts and lack necessary abstract and logical thinking abilities. Furthermore, a questionnaire survey administered to students who completed this course revealed that 52.68% found it difficult, while only 4.47% considered it easy or very easy.

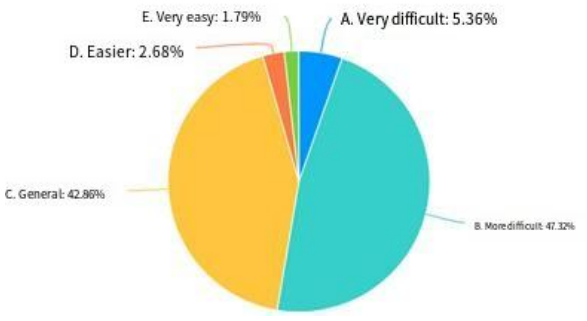

Figure 1: Survey Results Regarding Course Difficulty

Students exhibit a lack of intrinsic motivation and autonomy in learning, yet they are adept at utilizing fragmented time for study. On one hand, students demonstrate insufficient independent learning skills and tend to rely heavily on teachers. They require relearning under supervision and guidance due to their weak foundational knowledge in mathematics, low self-confidence, and diminished interest in the subject, which collectively hinder the development of sustained motivation. On the other hand, students excel at leveraging fragmented time for learning activities.

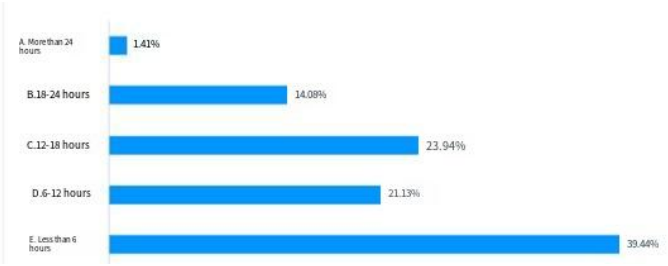

Figure 2: Survey Results of Students' Weekly Independent Learning Time

Students exhibit a strong inclination to apply their acquired knowledge to solve practical problems and frequently utilize short video platforms to gather information. However, they lack the skills necessary to effectively process and extract pertinent information, leading to diminished learning efficiency. Specifically, sophomore students demonstrate a robust willingness to apply learned knowledge to real-world issues after preliminary learning on short video platforms. Nonetheless, their inability to reprocess and critically analyze the obtained information results in suboptimal learning outcomes.

3. Challenges in Course Instruction

3.1 Overemphasis on Calculation, Neglect of Conceptual Understanding, and Disconnect from Practical Applications

In the study of Probability Theory and Mathematical Statistics, students encounter numerous concepts and formulas. Many calculations involve complex mathematical elements such as infinite series and integrals, which demand a solid mathematical foundation and strong abstract thinking skills. In classroom instruction, teachers often focus heavily on deriving mathematical formulas and demonstrating calculation techniques. Meanwhile, students frequently concentrate on memorizing various distributions and calculating probabilities, expectations, variances, and parameters of events. Consequently, they often lack a deep understanding of the fundamental concepts in the course, the underlying logical connections between different pieces of knowledge remain unclear, and their ability to apply this knowledge effectively is hindered.

3.2 Modeling poses significant challenges, the case studies are outdated, and the approach is disconnected from current professional requirements.

On one hand, the course encompasses numerous abstract concepts and principles, including random events, probability distributions, expectation, variance, parameter estimation, among others. To address practical problems in students' professional domains, these concepts must be effectively modeled by bridging "concrete phenomena" with the "fundamentals of probability and statistics." Students are required to identify real-world needs during the modeling process and optimize solutions based on actual conditions. This places high demands on students' analytical skills, problem-solving abilities, innovative thinking, independent thought, and hands-on practical capabilities. On the other hand, current teaching content in probability theory and mathematical statistics courses often relies heavily on classical theories. Although textbooks provide a wealth of case studies, the material is frequently outdated and disconnected from contemporary technologies, trends, and values. Consequently, this can limit students' perspectives and hinder their ability to understand and apply knowledge across different fields.

3.3 Insufficient assessment, low efficiency, and deviation from competency testing

The course assessment method relies on a single examination, which fails to comprehensively evaluate students' skills and practical application abilities, thereby affecting their enthusiasm for learning. Furthermore, due to students' relatively weak mathematical foundations, insufficient motivation, and limited independent learning abilities, they are more likely to encounter difficulties, lose interest and motivation in learning, and experience lower learning efficiency.

4. Investigation and Implementation of Teaching Reform

4.1 Strategy for Teaching Reform

Mathematician Halmos once stated that "problems are the heart of mathematics." Drawing on this insight and the educational psychology principle that "learning starts from the concrete," our course team has developed a teaching model combining "problem-oriented instruction with hierarchical case studies." Problem-oriented instruction can effectively stimulate students' interest in learning, promote cognitive development, deepen their understanding of mathematical concepts, and enhance their independent thinking skills. Through the selection, analysis, discussion, synthesis, and extension of cases, progressive case-based teaching guides students to engage in deeper inquiry and critical thinking. Consequently, this "problem-oriented + hierarchical case-based" teaching approach not only rapidly enhances students' learning interest and initiative but also fosters practical skills, innovation, teamwork, and communication abilities.

4.2 The Implementation Process of Teaching Reform

4.2.1. Restructuring Teaching Content

Reconstruction of teaching content is a critical component for enhancing teaching quality and student learning outcomes. In accordance with the course objectives and students' actual conditions, instructors will restructure the key concepts in each chapter following the pathway of "comprehending and memorizing knowledge - constructing a knowledge framework - transferring and applying knowledge." Additionally, a teaching approach that integrates "problem-oriented instruction + progressively complex case studies" will be adopted:

Table 1: Schematic Diagram of Reconstructed Course Content

The Content	**Definitions, concepts, theorems**	**Formulas, models**	**Practical application**
	(Understanding + memorization)	**(Build a body of knowledge)**	**(Migration and Application)**
The probability of random events	Sample space, random event, probability of random event, conditional probability, division of sample space, independence of event	Relations and operations of events, classical schemes, addition formula, subtraction formula, multiplication formula, total probability formula, Bayes formula	Relations and operations of events, classical schemes, addition formula, subtraction formula, multiplication formula, total probability formula, Bayes formula
One-dimensional random variables and their distributions	Random variable, discrete random variable, continuous random variable, distribution law, distribution function, probability density function	Six common distributions	Six common distributions
Multidimensional random variables	Two-dimensional discrete random variables, joint	Joint distribution law, edge distribution law	Joint distribution law, edge distribution law

and their distributions	distribution function, joint distribution law, edge distribution law		
Numerical characteristics of random variables	Mathematical expectation, variance, covariance, and correlation coefficient	Mathematical expectation, variance, covariance, and correlation coefficient	Mathematical expectation, variance, covariance, and correlation coefficient
Law of large numbers and Central limit theorem	Bernoulli's Law of large numbers, Hinchin's Law of large numbers, independent identically distributed Central limit theorem, de Moivre-Laplace central limit theorem	Chebyshev inequality, Central limit theorem	Chebyshev inequality, Central limit theorem
Sample and sampling distribution	Sample, statistic, Chi square distribution, t distribution, F distribution	Statistics	Statistics
Parameter estimates	Moment estimation, maximum likelihood estimation, evaluation criteria of estimators, interval estimation	Moment estimation, maximum likelihood estimation, evaluation criteria of estimators, interval estimation	Moment estimation, maximum likelihood estimation, interval estimation

4.2.2 Emphasis on Conceptual Understanding, Minimized Calculations, and Practical Application

For complex formulas, theorems, and concepts, a "problem chain + real-life cases" approach is employed to facilitate heuristic teaching. This method guides students to explore the unknown using known principles, study abstract concepts through concrete examples, and decompose complex wholes into manageable parts to better understand and master the material. For students who focus excessively on calculation skills, a "practical problems + software simulation cases" strategy is adopted. This approach helps students recognize that while software can easily handle calculations, solving real-world problems requires a deep understanding of probability and statistics concepts and mastery of the interrelationships between different areas of knowledge, enabling them to truly analyze and solve problems.

4.2.3 Multi-modeling, novel cases, and leading-edge applications

For complex modeling problems, a "scaffolding" strategy is employed to decompose practical challenges into multiple levels and stages, progressing from simple to complex and from basic to advanced. This approach guides students to independently complete the modeling process step by step. Additionally, when analyzing problems, instructors can engage students in discussions of well-established modeling examples to foster their modeling awareness. For outdated textbook cases, an inquiry-based learning method combining "current issues + cutting-edge cases" is adopted. This encourages students to focus on societal hot topics, understand the latest developments and trends in the field, thereby enhancing their social responsibility, broadening their perspectives, and improving their innovative and practical skills.

4.3 Restructuring the Teaching Process

The course team, leveraging the Super Star Learning Channel and Bilibili platform, designed a teaching model centered on "problem-oriented + progressive case studies". The detailed implementation process is outlined as follows:

Before class, the teacher assigns the learning tasks, and students independently study the relevant content using B station's short video platform. They then complete the pre-test in the Learning Pass.

In the class, the essence of probability and statistics was analyzed through a structured three-step approach: "posing questions + introducing cases -- analyzing problems + modeling cases -- solving problems + applying cases". This method enabled students to examine random phenomena and statistical problems from multiple perspectives, thereby enhancing their logical and abstract thinking skills through well-organized instruction.

After each class, students are required to complete a quiz within 24 hours to reinforce their comprehension of the concept formulas. Upon completing each chapter, students will undertake chapter-specific homework assignments to consolidate and apply the acquired knowledge in solving practical problems. Ultimately, students are encouraged to enhance their overall competencies, including practical and innovative skills, by participating in extracurricular activities such as the National Mathematical Contest in Modeling for College Students, Innovation and Entrepreneurship Competitions for College Students, and teacher-led projects.

4.4 Redesign and Enhance Online Teaching Resources

With the assistance of the Super Star Learning platform, the course team has developed a comprehensive database system comprising a main database, an exercise repository, and an assignment repository. Leveraging platforms such as QQ and Super Star Learning Pass, instructors can utilize fragmented time to promptly address students' questions. Additionally, through the Bilibili platform, we recommend students watch Song Hao's "Probability Theory and Mathematical Statistics" course. By integrating these diverse platforms, we have reconstructed online learning resources for students, enabling them to engage in learning anytime and anywhere using fragmented time, thereby enhancing learning efficiency.

4.5 Reevaluate Multiple Metrics

After several years of exploration, the course team reformed the single-assessment model into a multi-faceted evaluation system. (1) Leveraging the Super Star learning platform, we assess students' learning attitudes and outcomes, providing timely feedback on their performance. (2) Offline, students complete personalized chapter assignments that consist of two components: standard questions and open-ended questions. The standard questions, designed by instructors based on students' majors and classroom dynamics, test basic knowledge and application skills. The open-ended questions involve real-world cases related to the major, requiring students to collaborate in groups to apply chapter concepts. This approach prevents rote copying of answers from online sources, encouraging genuine engagement and critical thinking. It also evaluates students' ability to analyze and solve practical problems using acquired knowledge while fostering teamwork and innovation. (3) The final closed-book examination emphasizes dual assessment of both knowledge and practical application skills.

Table 2. Final Grading Criteria for Probability Theory

Total course score = online independent learning (5%) + offline class attendance and performance (5%) + after-class quiz (10%) + chapter homework (20%) + end-of-term closed paper test (60%)		
Teaching session	Assessment content	Scoring criteria
Online self-directed learning	Self-study of pre-class videos and teaching materials, as well as pre-class detection of learning release	5 points
Offline classroom attendance and performance	Attendance and class participation (mainly reflected by class interaction completion)	5 points
After-school quizzes	Study the after-school quizzes posted after each lesson	10 points

Chapter Large assignment	Open question type The relevance of the selected case to the content of this chapter	5 points
	The relevance and difficulty of the selected cases of the open question and the major	5 points
	The normativity and correctness of the answers to the conventional questions	10 points
Final closed book test	Knowledge + Ability	60 points

4.6 Incorporate Ideological and Political Elements Seamlessly

Probability Theory and Mathematical Statistics courses are intricately intertwined with real-life applications. The course team has identified and refined ideological and political elements, seamlessly integrating them into practical problems and modeling cases, as illustrated in Table 3 below. While delving into theoretical knowledge, the course emphasizes guiding students towards a correct worldview, life perspective, and values, enhancing logical reasoning skills, and fostering a sense of social responsibility, national pride, and familial affection. This approach ensures the comprehensive fulfillment of the educational mission to cultivate morality and nurture well-rounded individuals.

Table 3: ntegration of Thought and Politics into Real-World Problems through

Teaching content	Practical problems	Real cases	Ideological and political goals
Classical scheme	What is the probability of winning the first prize in the lottery?	Using classical probability to calculate the probability of first, second and third prizes in Chinese welfare Lottery two-color ball?	Rational decision making
Independence of event	Is the draw about order?	8 students in a dormitory want to watch the wonderful handball match in the Youth Games held in our school. But now there are only 3 tickets. Is it fair to draw lots to decide who will go to watch the match?	Establish the values of fairness and justice
Binomial distribution	1 percent hope, do 100 percent effort right?	Suppose a student has a 0.01 chance of passing an exam in probability and mathematical statistics. After he tries 400 times, he has a chance of passing at least once?	Set lofty ideals, be not afraid of difficulties, have the courage to try, and constantly go beyond ourselves
Bayes formula	Why does breach of contract affect personal credit investigation?	Let the lender for Xiao Wang the probability of keeping faith for the first time is 0.6, suppose the probability of Xiao Wang keeping faith for the first time to repay on schedule is 0.9, the probability of Xiao Wang not keeping faith for the first time to repay on schedule is 0.5. Solve the probability of Xiao Wang's repayment on time.	Enhance students' awareness of integrity, improve law-abiding and rule-abiding consciousness, firmly establish the moral concept of honor and shame of breaking promises, and aspire to be a person who is honest and ethical since childhood.

Mathematical expectation	Is the 10-person mixed test implemented during the epidemic justified?	If the positive probability of samples in a community of batch confirmed cases in a certain area is 1%, how reasonable is it to carry out multiple rounds of 10-person mixed testing?	To stimulate students' confidence and pride in their country
Law of Large Numbers	Why is the casino sure to win?	Take a special gambling device - roulette as an example, roulette is divided into 37 equal grids, corresponding numbers 0-36, the beginning of the game will pop out a ball, at the same time the roulette began to rotate, when the roulette stops, the small ball will fall in a grid, then according to the play of betting singles and doubles, if the ball falls into the singular, then the singular grid chips double, bet double money to the casino. 0 as a special number, as long as it falls into 0, no matter the bet single bet double, the money belongs to the casino, please use the law of large numbers to explain why the casino is stable and does not lose?	Set up the right values, do not report gambler mentality
Central limit theorem	Why do airlines always overbook?	An airline has a plane with 300 seats. On a given day, the airline sells 320 tickets. Assuming that each passenger acts independently, and the probability of getting on the flight as scheduled is 0.9, what is the probability that the plane will have enough seats?	Learn to apply

5. Impact of Reforms

5.1 The development of course construction and the establishment of the course team have yielded preliminary outcomes.

The course team has undertaken extensive curriculum development for probability theory and mathematical statistics courses. Specifically, they have established a school-level undergraduate model course, initiated a school-level teaching reform project, implemented a school-level curriculum ideological and political demonstration project, secured a school-level teacher development fund, and developed a school-level gold course. Additionally, the team has achieved notable accolades, including a third prize in the inaugural school-level innovation competition, second prizes in the third and fourth school-level innovation competitions, a provincial first prize in the Youth Education Competition, and a third prize in teaching design.

5.2 Significant Student Recognition

A comparison of the questionnaire survey data before and after the reform reveals that students' interest in courses has significantly improved, with the proportion of students who are very interested in courses increasing by 22.14%. Post-innovation teaching saw a substantial rise in homework frequency from once per chapter to after every class. Notably, 55% of students found this increased homework load to be highly reasonable, marking a 52% increase from the pre-reform period. Furthermore, 52.63% of students rated the classroom teaching method as very effective, while 38.16% felt that the course greatly enhanced their mathematical thinking skills. Students also expressed high satisfaction with the case studies selected following the curriculum reform.

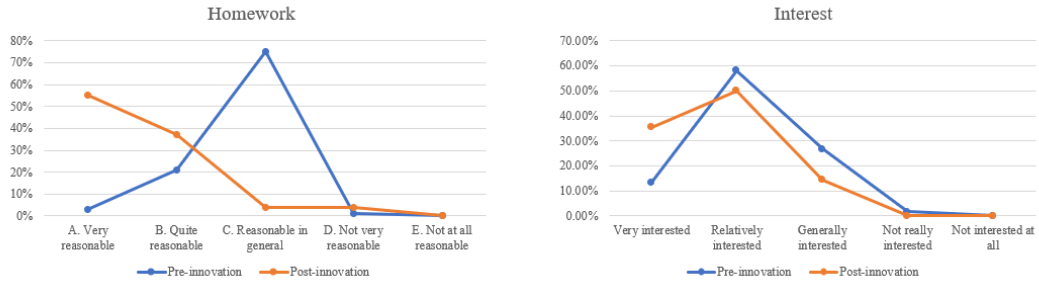

Figure 4: Summary of Student Feedback Results

5.3 Students achieve significant benefits

Since the implementation of the teaching reform, our team's teacher training program has successfully guided students to participate in various competitions. In the National Mathematical Modeling Competition for college students, we have won 1 national second prize, 5 district first prizes, 8 district second prizes, and 24 district third prizes. In the "Internet +" College Students Innovation and Entrepreneurship Competition, our achievements include 1 gold award and 8 bronze awards. Additionally, in the National College Students Mathematics Competition, our students have achieved 8 first prizes, 10 second prizes, and 50 third prizes.

6. Reflection and Enhancement

Application cases can be further developed and categorized, emphasizing the latest advancements in various disciplines to better align with the specific needs of students from different majors. The integration of ideological and political elements into the cases requires further supplementation and expansion.

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