A Novel Axiomatic Design-based Method for Membrane Material Supplier Selection Considering Group Consensus

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Abstract:

This paper proposes an axiomatic design-based mathematical programming method for membrane material supplier selection with real number and linguistic term, in which the weights of experts and attributes are unknown. Firstly, this paper proposes a new information measure based on axiomatic design and defines the information content of linguistic term set. Then, based on the assumption that the attribute weights with respect to each expert are different, this paper constructs a mathematical programming model through maximizing the deviation to determine the attribute weights. Subsequently, considering the differences among different experts, this paper constructs a mathematical programming model to derive experts' weights by maximizing group consensus. The group information content of each alternative is calculated to determine the ranking order of alternatives. Finally, the proposed method is applied to solve the membrane material supplier selection problem for verifying its practicability and effectiveness. Besides, the proposed method is compared with other methods to show its advantages.

Keywords: Axiomatic design; Heterogeneous multi-criteria group decision making; Linguistic term set; Group consensus; Sustainable supplier selection

1. Introduction

Membrane material supplier selection can be regarded as a multi-attribute group decision-making (MADM) problem. MADM is a tough decision-making process, in which the decision-maker (DM) requires to identify the most satisfactory one from multitudinous alternatives according to some conflicting evaluation indicators (attributes or criteria). MADM has become an important part of modern decision-making theory. For example, when an enterprise DM would like to select a material supplier, he/she should consider multiple attributes, such as cost, quality, carbon dioxide emission and information disclosure, etc. The attributes "cost " and "quality" generally conflict with each other. In addition, the attribute values are usually represented by different types of information. For example, real numbers can be used to express the cost, and linguistic terms (e.g., "better" or "worse") are used to describe quality. At the same time, different DMs often pay different attention to different attributes, which means that different DMs would like to assign different weights for different attributes. In this paper, the heterogeneous multi-attribute group decision-making (HMAGDM) problem is defined by the MAGDM problem that contains more than two types of evaluation information and allows DMs to freely set attribute weights. However, the traditional MAGDM method failed to consider the difference among DMs' requirements and the diversity of different decision-making information, which might limit the application of the traditional MAGDM method. It is inconsistent with the actual decision-making problems in real life. Therefore, the HMAGDM problem considered in this paper can effectively make up the shortcomings of traditional MADM methods.

Generally, the traditional MADM method usually employs crisp numbers to evaluate each alternative under each attribute. However, due to the complicity of the decision information and the uncertainty of human thinking, it is insufficient for crisp numbers to express the nature of decision problems and the true intention of DMs. To cover this issue, Prof. Zadeh [1] pioneered fuzzy sets theory to describe the uncertainty or fuzziness of the decision problems. Henceforth, numerous types of fuzzy information are widely-used to solve the real decision problems, such as intervals [2], triangle fuzzy numbers (TFNs) [3], trapezoidal fuzzy numbers (TrFNs) [4], intuitionistic fuzzy numbers (IFNs) [5] and linguistic terms (LTs) [6]. Besides, in order to better reflect the difference among DMs, DMs are permitted to freely assign the attribute weights. In this paper, the decision-making problem that has different types of fuzzy information and allows DMs to freely assign the attribute weights is called HMAGDM. In recent years, HMAGDM has attracted many scholars' attention and produced some valuable research results. For example, Yu et al. [7] extended TOPSIS (Technique for Order Preference by Similarity to an Ideal Solution) to solve the decision-making problem with real information and linguistic information; Feng and Lai [8] proposed a comprehensive method to handle China Southern Airlines' freight forwarder selection problem, which takes into account the psychological behavior of DMs; Dong and Zhang [9] proposed a consensus framework for multi-person decision-making problems with different preference representation structures (preference order, utility function, reciprocal preference relation and fuzzy preference relation); Chao et al. [10] developed a consensus-reaching model to solve heterogeneous large-group decision-making with non-cooperative behavior, in which cosine similarity is introduced to construct a distance measure for different preference structure.

From the perspective of how to measure the differences between alternatives, the existing HMAGDM methods can be roughly divided into three categories: distance-based methods [11][12][13], relative closeness-based methods [14][15][16][17][18] and psychological terminology-based methods [8][19][20].

The first category of HMAGDM used different types of distance tool to measure the differences between alternatives. For example, Liao et al. [11] used Euclidean distance to represent the difference between attribute values, and established a mathematical programming model to derive the attribute weight. Wang et al. [12] converted real numbers and linguistic variables into interval-valued intuitionistic fuzzy numbers and used Euclidean distance to define group consistency and inconsistency indexes. Zhang et al. [13] used Hamming distance to measure the difference between alternatives and determined the weight of the best attribute by building the maximum deviation model.

The second category of HMAGDM used relative closeness to measure the measure the differences between alternatives. For example, Haje et al. [14] applied the TOPSIS to the interval-valued intuitionistic fuzzy environment. They proposed a novel MAGDM method to solve the supplier selection problem by considering the interaction between the criteria. Li and Wan [21] proposed a HMAGDM method, which ranks the alternatives by calculating the comprehensive relative closeness and the weights of experts are given in advance. Lourenzutti and Krohling [15] proposed a TOPSIS-based HMAGDM method in a dynamic environment where the experts of criteria and experts are known. Li et al. [16] used a weighted average operator to integrate heterogeneous information and ranked alternatives by similarity. Yu et al. [17] proposed a HMAGDM method with various types of evaluation values, which analyzed the relative closeness between the alternative and the positive ideal solution through the onress measure. Zhang and Xu [18] developed a fuzzy TOPSIS-based maximizing consensus method to solve MAGDM problem and ranked alternatives by relative closeness.

The third category of HMAGDM introduced DMs' psychological behaviors. It is obvious that DMs' psychological behaviors play an important role in the process of decision-makings. Hence, some scholars introduced the DMs' psychological behaviors to measure the difference between alternatives. Currently, the frequently-used representations of psychological behaviors are expected utility function [8] and prospect values [19][20]. Zhang et al. [22] proposed a bi-objective intuitionistic fuzzy programming model, which used comprehensive prospect values to define consistency and inconsistency index.

Although the above-mentioned methods can solve some specific heterogeneous group decision problems, they still have the following shortcomings:

(1) Traditional MAGDM methods used distance or relative closeness to measure the difference between candidates, but sometimes this difference is difficult to distinguish [27]. The distance (relative closeness or psychological terms) based methods are hard to distinguish the difference when they are used to measure the difference between alternative and PISs (positive ideal solutions).

(2) The existing methods assumed that the weights of attributes/experts are equal or completely known, which is inconsistent with actual decision-making. The pre-given weights of experts surely result in some subjective arbitrariness. Although methods [7][20][23][24] derived experts' weights by building some programming models, they assumed that all experts assign the same weight on all criteria, which failed to consider experts' different cognitions on different criteria.

(3) Due to the different attribute dimensions, most of the existing methods need to normalize the evaluation information, which surely results in serious information loss

To cover the above shortcomings, this paper proposes a HMAGDM method considering group consensus based on axiomatic design theory. The main contributions of this paper are summarized as follows:

(1) Aiming at the first shortcoming, this paper uses the information content to measure the linguistic term based on axiomatic design and ranks the alternatives by the comprehensive information content. This is the biggest innovation of the paper. The information content definition for linguistic term is initially provided.

(2) Based on the assumption that the attribute weights with respect to each DM are different, this paper constructs a mathematical programming model through maximizing the deviation to determine the attribute weights. Considering the differences among different experts, this paper constructs a mathematical programming model to derive experts' weights by maximizing group consensus.

(3) The method proposed in this paper directly uses the original information and does not carry out standardized processing. It retains the original information to the greatest extent.

The layout of this paper is conducted below. Section 2 describes the HMAGDM problem and defines some mathematical symbols. Section 3 quantitates the linguistic terms. Section 4 proposes axiomatic design-based HMAGDM method. In Section 5, the proposed method is applied to solve a sustainable supplier selection problem. This paper ends with some conclusions and research prospects in Section 6.

2. Problem description

At present, the studies on HMAGDM mainly use the quantitative information, such as real numbers and intervals. Quantitative information limits the application of these studies to some real-life decision-making problems. In real decision-makings, due to the uncertainty and fuzziness of human thinking, DMs may express their evaluations in the form of linguistic terms and real numbers. Therefore, it is of great theoretical and practical significance to study the decision making problems whose attribute values are real numbers and linguistic terms.

To facilitate the subsequent work, some mathematical symbols of the studied HMAGDM are defined as follows:

(1) $L=\{1,2,\ldots,l\}$, $M=\{1,2,\ldots,m\}$ and $N=\{1,2,\ldots,n\}$ are three index sets.

(2) $A=\{a_1,a_2,...,a_m\}$ is the alternative set.

(3) $C=\{c_1,c_2,...,c_n\}$ is the attribute set.

(4) $E=\{e_1,e_2,\ldots,e_l\}$ is the DM set.

(5) $\boldsymbol{w}^k=(w_1^k,w_2^k,\cdots,w_n^k)^{\mathrm{T}}$ with $\sum_{j=1}^{n} w_j^k=1$ and $w_j^k\in[0,1]$ is the attribute weight vector with respect to DM e_k.

(6) $\boldsymbol{\lambda}=(\lambda_1,\lambda_2,...,\lambda_l)^{\mathrm{T}}$ with $\sum_{k=1}^{l}\lambda_k=1$ and $\lambda_k\in[0,1]$ is the DM weight vector.

(7) $$R=(r_{ij}'^{k})_{m\times n}=\begin{array}{c|c} & \begin{array}{cccc} c_1 & c_2 & \cdots & c_n \end{array} \\ \begin{array}{c} a_1 \\ a_2 \\ \vdots \\ a_m \end{array} & \begin{pmatrix} r_{11}'^{k} & r_{12}'^{k} & \cdots & r_{1n}'^{k} \\ r_{21}'^{k} & r_{22}'^{k} & \cdots & r_{2n}'^{k} \\ \vdots & \vdots & \vdots & \vdots \\ r_{m1}'^{k} & r_{m2}'^{k} & \cdots & r_{mn}'^{k} \end{pmatrix} \end{array} (k=1,2,\cdots,l)$$ is the initial evaluation matrix given by DM e_k.

3. Information content of linguistic terms and real numbers

The quantification of LTs is the focus of qualitative MADM methods. Linguistic term is a decision-making tool to express the qualitative preference information of DMs/experts. It includes two indicators: linguistic scales and the corresponding semantics. There are many methods to define different linguistic scales and semantics. A common method is the linguistic term set based on the ordered structure method. At present, the quantitative methods of LTs mainly include: linguistic symbol model-based method [25], binary semantic model-based method [26], and cloud model-based method [28]. Linguistic symb ol model-based method and binary semantic model-based method converts the LTs into real numbers, which ignores the fuzziness and uncertainty in qualitative evaluation. Li et al. [28] put forward the framework of cloud model in 1995, which can simultaneously reflect the randomness and fuzziness of qualitative concepts. Although cloud model is an effective tool to describe qualitative concepts, it is limited by the complex processing.

Axiomatic design has been widely applied to MAGDM in recent studies. In this paper, based on the concept of axiomatic design, a new quantitative method of LTs is proposed. Firstly, the "design scope" and "system scope" in the axiomatic design are regarded as the ideal alternative and evaluation alternative of the MAGDM problem respectively. Then, the closeness between the ideal alternative and evaluation alternative are measured based on information content. Lastly, the alternatives are ranked according to the closeness. In this method, the "design scope" and "system scope" in axiomatic design can better describe the uncertainty of LTs.

In order to better describe the uncertainty of LTs, LTs are first transformed into corresponding TFNs according to certain rules. Let $S=\{s_i \mid i=0,1,\cdots,g\}$ be a set of LTs containing $g+1$ (g is an even number) linguistic variables, where s_i represents a language variable. The linguistic variables are transformed into corresponding triangular fuzzy numbers through function ∇. The definition of conversion function ∇ is $\nabla(s_i)=\tilde{m}=(m_1,m_2,m_3)$, where $m_1=\frac{F(s_i)}{2g}$, $m_2=\frac{0.5g+F(s_i)}{2g}$ and $m_3=\frac{g+F(s_i)}{2g}$. $F(s_i)$ is the scale function proposed by Wang et al. [29].

Let s_α and s_β be the "design scope" and "system scope" in axiomatic design, respectively. Then, s_α and s_β can be converted into TFNs by the following formulas.

$$\nabla(s_\alpha)=(m_1,m_2,m_3)=(\frac{F(s_\alpha)}{2g},\frac{0.5g+F(s_\alpha)}{2g},\frac{g+F(s_\alpha)}{2g}),$$

$$\nabla(s_\beta)=(n_1,n_2,n_3)=(\frac{F(s_\beta)}{2g},\frac{0.5g+F(s_\beta)}{2g},\frac{g+F(s_\beta)}{2g})$$

The information content between s_α and s_β is calculates as:

$$I(s_\alpha,s_\beta)=\begin{cases}\log_2\frac{g^2}{(g+F(s_\alpha)-F(s_\beta))^2}, & g+F(s_\alpha)>F(s_\beta)\\ \infty & , g+F(s_\alpha)\le F(s_\beta)\end{cases} \tag{1}$$

Example 1 Let $S=\{s_\alpha \mid \alpha=0,1,\cdots,6\}$ be a set of seven LTs. If $F(s_i)=\eta_i=i/2t$, then the LTs and the corresponding TFNs are determined and shown in Table 1.

Table1. LTs and their corresponding TFNs.

LTs	TFNs
S_0=Absolutely low (AL)	(0.00, 0.25, 0.5)
S_1=Very low (VL)	(0.08,0.33,0.58)
S_2=Low (L)	(0.17,0.42,0.67)
S_3=Medium (M)	(0.25,0.50,0.75)
S_4=High (H)	(0.33,0.58,0.83)
S_5=Very high (H)	(0.42,0.67,0.92)
S_6=Absolutely high (AH)	(0.50,0.75,1.00)

The conversion approach proposed by this paper can ensure that there exist overlapping between any two TFNs (shown in Fig. 1), which can better describe the fuzziness of LTs. For example, let s_4 and s_5 be the "design scope" and "system scope". According to Eq. (2), the information content between s_4 and s_5 is calculates as:

$$I(s_4,s_5)=\log_2\frac{36}{(6+F(s_4)-F(s_5))^2}=\log_2\frac{36}{(6+4-5)^2}=\log_2\frac{36}{25}$$

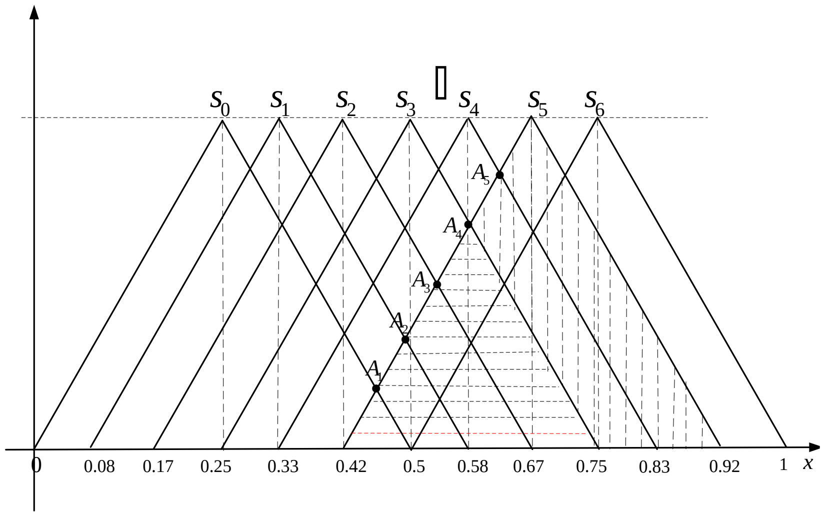

Figure 1. Linguistic variables of overlapping 7-point scales.

If the system range and design range are represented by two real numbers q and q^*, respectively. The information content is defined as follows[27]:

$$I(q,q^*) = \log_2(q^* / q) \quad (q \le q^*) \tag{2}$$

4. HMAGDM method based on axiomatic design

In order to solve the MAGDM problem described in Section 2, this section proposes a HMAGDM method based on axiomatic design. Firstly, this paper constructs a maximum deviation model to determine the weights of attributes with respect to each expert. Then, considering the differences between experts, this paper proposes a maximum group consensus-based method to determine the weights of experts. Finally, the detailed steps of this method are summarized.

4.1 Determine the weights of attributes

Due to the complexity of the decision-making environment and the limited knowledge of DMs, the attribute weight information of MAGDM problems in real life is usually pregiven, which is very arbitrary. However, the attribute weights will affect the ranking results of alternatives. Therefore, how to scientifically determine the attribute weights is a problem that must be solved in the MAGDM problem. At present, most MAGDM methods assume that all experts give evaluation values under a given index system and attribute weight information, which is obviously inconsistent with the actual situation. It is unrealistic for the DMs with different educational backgrounds, knowledge and experience to give the same attribute weights. For example, expert A thinks the first attribute should be assigned the maximum weight, while expert B deems the second attribute should be assigned the maximum weight. This section intends to construct a maximum deviation optimization model to solve such kind of problem.

The core idea of the maximum deviation method is described as follows: if there exists a large difference among the evaluation values of all alternatives under an attribute, then this attribute plays an important role in ranking alternatives, which indicates that this attribute should be given a larger weight, and vice versa. Different from the distance-based measurement used in traditional MAGDM methods, this paper adopts information content to measure the differences among the alternatives.

Let I_{ij}^k be the information content of alternative a_i under attribute c_j given by e_k. The deviation between I_{ij}^k and I_{tj}^k is defined as follows:

$$D_{ij}^k = \frac{2}{m(m-1)} \sum_{t=i+1}^{m} w_j^k \, | I_{ij}^k - I_{tj}^k | \quad (i \in M; j \in N; k \in L) \tag{3}$$

Then, the total deviation of any two alternatives' information content under attribute c_j given by e_k can be formulated as follows:

$$D_j^k = \frac{2}{m(m-1)}\sum_{i=1}^{m-1} D_{ij}^k = \frac{2}{m(m-1)}\sum_{i=1}^{m-1}\sum_{t=i+1}^{m} w_j^k \,|\, I_{ij}^k - I_{tj}^k \,| \quad (i \in M; j \in N; k \in L) \tag{4}$$

According to the deviation maximization method [35], this paper intends to maximize the total deviation to determine the weights of attributes. Based on this, the following programming model can be built to determine the weights of attributes.

$$\max \min_{j \in N} D_j^k = \frac{2}{m(m-1)}\sum_{i=1}^{m}\sum_{t=i+1}^{m} w_j^k \,|\, I_{ij}^k - I_{tj}^k \,|$$
$$s.t.\begin{cases} \sum_{j=1}^{n} w_j^k = 1 \\ w_j^k \geq 0 \quad (j = 1, 2, \cdots, n) \end{cases} \tag{5}$$

Let $\eta^k = \min\{D_1^k, D_2^k, \cdots, D_n^k\}$. Then, model (5) can be transformed into the following single-objective model.

$$\max \eta^k$$
$$s.t.\begin{cases} \sum_{i=1}^{m}\sum_{t=i+1}^{m} w_j^k \,|\, I_{ij}^k - I_{tj}^k \,| \geq \eta^k \ (j = 1, 2, \cdots, n) \\ \sum_{j=1}^{n} w_j^k = 1 \\ w_j^k \geq 0 \quad (j = 1, 2, \cdots, n) \end{cases} \tag{6}$$

Obviously, model (6) can be easily solved by some software. Then, the attribute weight vector with respect to e_k, i.e., $\boldsymbol{w}^k = (w_1^k, w_2^k, \cdots, w_n^k)^{\mathrm{T}}$, can be determined.

4.2 Determine the weights of experts

Because different DMs have different importance in group decision-making, it is logical to assign different weights to DMs in real-life MAGDM. In addition, the high the group consensus among DMs, the more reliable the group decision-making. Based on this, this paper introduces the group consensus to derive the weights of DMs.

Based on the obtained weight vector $\boldsymbol{w}^k = (w_1^k, w_2^k, \cdots, w_n^k)^{\mathrm{T}}$, the information content of alternative a_i (denoted by I_i^k) can be expressed as:

$$I_i^k = \sum_{j=1}^{n} w_j^k I_{ij}^k \tag{7}$$

Let $\boldsymbol{\lambda} = (\lambda_1, \lambda_2, ..., \lambda_l)^{\mathrm{T}}$ be the weight vector of DMs. Then, the total information content of alternative a_i can be calculated as:

$$I_i = \sum_{k=1}^{l} \lambda_k w_j^k I_i^k = \sum_{k=1}^{l} \lambda_k \sum_{j=1}^{n} w_j^k I_{ij}^k \tag{8}$$

Let DI_i^k be the deviation between the total information contents of alternative a_i given by individual expert e_k and expert group. Then, the deviation with respect to e_k is defined as:

$$DI_i^k = |\, I_i^k - I_i \,| \ (i \in M; k \in L)$$

The consensus index between DM e_k and group with respect to alternative a_i is defined as:

$$CI_i^k = 1 - |\, I_i^k - I_i \,|$$

Obviously, CI_i^k has the power to measure the consensus level of e_k and the group. Generally, the larger the value of CI_i^k, the higher the consensus level of e_k.

The consensus level of e_k with respect to all alternatives can be calculated as:

$$CI^k = \sum_{i=1}^{m}(1-|I_i^k - I_i|) \quad k=1,2,\cdots,t$$

Let CI be the group consensus level. Obviously, the calculation formula of CI is shown as:

$$CI = \sum_{k=1}^{l}\sum_{i=1}^{m}(1-|I_i^k - I_i|) \tag{9}$$

Clearly, Eq. (9) can be rewritten as:

$$CI = lm - \sum_{k=1}^{l}\sum_{i=1}^{m}|I_i^k - I_i| \tag{10}$$

The larger the value of CI, the higher the group consensus level. In order to determine the weights of DMs $\lambda_k\ (k=1,2,\cdots,l)$, this paper constructs the following programming model:

$$\max CI = lm - \sum_{k=1}^{l}\sum_{i=1}^{m}|I_i^k - I_i|$$
$$s.t.\begin{cases}\sum_{k=1}^{l}\lambda_k = 1\\ \lambda_k \ge 0 \quad (k=1,2,\cdots,l)\end{cases} \tag{11}$$

In order to solve the model (11) conveniently, denote $\tau_i^{k+} = \frac{1}{2}(|I_i^k - I_i| + I_i^k - I_i)$ 和 $\tau_i^{k-} = \frac{1}{2}(|I_i^k - I_i| - I_i^k + I_i)$. Then, it holds that $|I_i^k - I_i| = \tau_i^{k+} + \tau_i^{k-}$ and $I_i^k - I_i = \tau_i^{k+} - \tau_i^{k-}$. Model (11) can be converted into the following linear programming model.

$$\min \sum_{k=1}^{l}\sum_{i=1}^{m}(\tau_i^{k+} + \tau_i^{k-})$$
$$s.t.\begin{cases}\tau_i^{k+} - \tau_i^{k-} = I_i^k - I_i \ (i=1,2,\cdots,m;\ k=1,2,\cdots,l)\\ \sum_{k=1}^{t}\lambda_k = 1,\ \ \lambda_k \ge 0 \quad (k=1,2,\cdots,l)\\ \tau_i^{k+}, \tau_i^{k-} \ge 0 \ (i=1,2,\cdots,m;\ k=1,2,\cdots,l)\end{cases} \tag{12}$$

According to Eq. (8), model (12) can be transformed into model (13).

$$\min \sum_{k=1}^{l}\sum_{i=1}^{m}(\tau_i^{k+} + \tau_i^{k-})$$
$$s.t.\begin{cases}\tau_i^{k+} - \tau_i^{k-} = I_i^k - \sum_{k=1}^{l}\lambda_k I_i^k \ (i=1,2,\cdots,m;\ k=1,2,\cdots,l)\\ \sum_{k=1}^{t}\lambda_k = 1,\ \ \lambda_k \ge 0 \quad (k=1,2,\cdots,l)\\ \tau_i^{k+}, \tau_i^{k-} \ge 0 \ (i=1,2,\cdots,m;\ k=1,2,\cdots,l)\end{cases} \tag{13}$$

The weight vector of DMs $\boldsymbol{\lambda} = (\lambda_1, \lambda_2, \cdots, \lambda_l)^{\mathrm{T}}$ can be determined by solving model (13).

4.3 Steps of HMAGDM method based on Axiomatic design

After the weights of attributes and DMs are obtained, the total information content of each alternative can be obtained by weighted aggregation approach. Then, the alternatives can be ranked according to their information content and the best alternative can be determined. Next, we conclude the he detailed steps of the HMAGDM method considering group consensus.

Step 1. Construct the decision matrix $R^k = (r'^k_{ij})_{m\times n}$, where r'^k_{ij} means the evaluation value of alternative a_i under attribute c_j given by DM e_k.

Step 2. According to Eqs. (1) and (2), the information content of all evaluation values can be calculated.

Step 3. The attribute weight vector $\boldsymbol{w}^k = (w_1^k, w_2^k, \cdots, w_n^k)^{\mathrm{T}}$ can be determined by solving model (6).

Step 4. The weight vector of DMs $\boldsymbol{\lambda} = (\lambda_1, \lambda_2, \cdots, \lambda_l)^{\mathrm{T}}$ can be determined by solving model (13).

Step 5. The total information content of all alternatives can be calculated by using Eq. (8).

Step 6. The alternatives can be ranked according to their total information content. The smaller the total information content, the better the alternative.

5. Case study on sustainable supplier selection

To show the practicability of the proposed HMAGDM method, it is used to solve membrane material supplier selection. Besides, the proposed HMAGDM method is compared with other similar methods to verify its advantages.

5.1 Case background description

A membrane manufacturing company is looking for material supplier. Sustainable membrane material suppliers are of significant importance to membrane manufacturing companies, such as ensuring supply chain stability, enhancing product quality, and reducing costs. It also can foster exchange and cooperation in material research and development, process improvement, and other areas. The partnership is conducive to technological innovation and product upgrades for membrane technology manufacturers. Therefore, sustainable supplier selection becomes a critical decision-making issue. It can be solved by the proposed method in this paper. After brainstorming group meeting, the enterprise establishes an evaluation system of 13 indicators shown in Table 2.

Table 2. Sustainability evaluation index and its meaning.

Attribute type	Attribute	Meaning
Economy	Cost (C_1)	Product price, purchase cost, holding cost and ordering cost
	Quality (C_2)	Product quality and reliability
	Delivery (C_3)	Ability and reliability of on-time delivery
	Service (C_4)	After-sales service level
	Flexibility (C_5)	Flexibility level of materials supplied and price discount of materials supplied
Environment protection	Technical level (C_6)	New technology production capacity
	Environmental Management System (C_7)	A systematic process that enables the supplier to reduce the environmental impact
	Resource consumption (C_8)	Reduce the use of energy, electricity, water and other resources by improving production, maintenance and process, saving, recycling and reusing materials and other measures
	Environmental design (C_9)	Consider environmental impact when designing products throughout the product life cycle (including procurement, manufacturing, use and disposal stages)
	Reduce, reuse and recycle (C_{10})	Reduce pollution (such as air pollution and water pollution), green packaging and waste recycling
Society	Occupational health and safety (C_{11})	Implement relevant regulations (such as OHSAS 18001) to ensure labor safety and health (including physical and mental health)
	Employee rights and benefits (C_{12})	Focus on relevant factors and needs of employees to achieve long-term sustainable effectiveness
	Information disclosure (C_{13})	Provide customers and stakeholders with information on materials and carbon emissions used in the production process

After careful screening, ten suppliers are finally selected as candidate suppliers (i.e., alternatives). For convenience, these alternatives are denoted by a_i $(i = 1, 2, \cdots, 10)$. In order to effectively evaluate these alternatives, the company selects four department managers from different departments as DMs. They are strategic procurement manager (e_1), design department manager (e_2), production department manager (e_3) and quality control manager (e_4). Obviously, the expertise of these DMs is different, which is convenient for gathering group wisdom and making scientific and reasonable evaluation. DMs use precise numbers and LTs to express their preferences based on their professional knowledge and suppliers' historical performance. The evaluation values of alternatives in terms of sustainability assessment attributes given by four DMs are shown in Tables 3-6.

5.2 Decision-making process

Step 1. Through field visits, the four DMs give corresponding evaluation information based on their professional knowledge and work experience, as shown in Tables 3-6.

Table 3. Evaluation value of expert $e_{1.}$

	C1	C2	C3	C4	C5	C6	C7	C8	C9	C10	C11	C12	C13
A1	68	VG	M	MP	MG	M	VG	MP	P	G	MG	VP	P
A2	87	M	MG	VP	P	VG	MP	GP	M	VG	MP	MG	G
A3	72	MP	VG	M	G	MP	MG	P	G	P	M	VP	VG
A4	61	MG	M	G	VG	P	G	M	MP	P	MG	MP	VP
A5	83	G	P	VG	M	VG	MP	MG	G	P	M	VP	MG
A6	89	MP	P	MP	G	VG	G	VG	M	MG	VP	MG	M
A7	67	P	VG	M	MG	MP	P	MG	M	MP	G	VG	VP
A8	77	MG	VP	G	P	M	VG	MP	MG	VG	MP	P	G
A9	95	VG	MG	M	VG	G	MP	M	P	MP	G	P	MG
A10	68	MG	MP	VG	MP	M	P	G	VG	MG	P	M	VP

Table 4. Evaluation value of expert $e_{2.}$

	C1	C2	C3	C4	C5	C6	C7	C8	C9	C10	C11	C12	C13
A1	62	G	MP	MG	VG	M	MP	G	MG	M	P	VP	P
A2	88	VP	M	VG	P	MG	G	MP	M	VG	G	MG	MP
A3	73	MP	P	MG	G	M	MG	VP	VG	MP	M	G	P
A4	78	MG	VG	MP	MG	VG	G	M	G	P	MP	VP	P
A5	84	VG	M	MG	G	P	M	M	VP	MG	P	MP	P
A6	88	MP	P	MP	M	G	MG	G	M	MG	VP	MG	VG
A7	68	M	MP	G	P	MG	P	MP	VG	MG	MG	VP	VG
A8	89	G	MG	G	P	VG	G	MG	M	MP	VP	M	MP
A9	73	VG	MP	P	G	MG	G	VP	M	VG	MP	P	M
A10	77	MP	VP	MG	P	G	M	VG	MP	P	G	MG	VG

Table 5. Evaluation value of expert $e_{3.}$

	C1	C2	C3	C4	C5	C6	C7	C8	C9	C10	C11	C12	C13
A1	77	G	VG	MP	G	MG	VG	MP	M	MG	P	VP	P
A2	67	P	G	VP	MP	MG	M	VG	MG	P	MP	M	VG
A3	89	MG	G	MG	M	VG	P	G	MP	M	VG	MP	VP
A4	83	VG	MG	VG	MP	M	G	M	MG	G	P	VP	P
A5	62	MP	VP	MP	VG	M	P	MG	G	M	MG	G	P
A6	97	M	MP	MG	M	MG	VG	G	P	MP	G	VG	P
A7	78	MG	P	M	G	P	MG	MP	VG	VP	VG	MP	G
A8	89	G	M	MP	MG	VG	G	VG	MG	M	MP	P	VP
A9	83	P	G	MG	G	VP	M	MG	MP	VG	M	P	VG
A10	72	M	VG	G	MG	VG	MP	M	P	VP	G	MP	P

Table 6. Evaluation value of expert e_4.

	C1	C2	C3	C4	C5	C6	C7	C8	C9	C10	C11	C12	C13
A1	67	MP	MG	VG	G	M	MG	P	VG	M	VP	P	MP
A2	96	P	MP	G	VG	M	G	VG	MG	MP	MG	M	P
A3	82	VG	P	M	P	G	M	MG	VP	G	MP	MG	VG
A4	88	M	P	G	MG	M	MP	G	MG	VG	MP	VG	VP
A5	92	G	MP	MG	P	VG	MG	VG	G	M	MP	P	M
A6	68	P	VG	VG	MG	M	MP	G	MG	MP	M	P	VP
A7	88	M	G	MP	P	M	VG	MG	VP	MG	VG	MP	G
A8	93	P	M	MG	MP	MG	MP	G	VG	M	G	VG	P
A9	64	MG	G	MP	VG	MG	G	P	M	P	MP	VP	M
A10	61	VP	P	MG	MP	VG	M	MG	P	G	M	MP	G

Step 2. The evaluation information shown in Tables 3-6 can be converted into the corresponding information content, as shown in Tables 7-10. It is worth noting that most of the existing methods need to normalize the evaluation information, which surely results in serious information loss. However, it is no need for the information content-based approach to normalize the evaluation information, which can reduce the information loss effectively.

Table 7. Information content of expert e_1.

	A1	A2	A3	A4	A5	A6	A7	A8	A9	A10
C1	0.7548	0.1986	0.6258	1.0000	0.3048	0.1473	0.7882	0.4742	0.0000	0.7548
C2	0.0000	0.3869	0.6131	0.2263	0.1018	0.6131	1.0000	0.2263	0.0000	0.2263
C3	0.3333	0.1950	0.0000	0.3333	0.8617	0.8617	0.0000	1.0000	0.1950	0.5283
C4	0.5283	1.0000	0.3333	0.0877	0.0000	0.5283	0.3333	0.0877	0.3333	0.0000
C5	0.2263	1.0000	0.1018	0.0000	0.3869	0.1018	0.2263	1.0000	0.0000	0.6131
C6	0.3869	0.0000	0.6131	1.0000	0.0000	0.0000	0.6131	0.3869	0.1018	0.3869
C7	0.0000	0.6131	0.2263	0.1018	0.6131	0.1018	1.0000	0.0000	0.6131	1.0000
C8	0.6131	0.1018	1.0000	0.3869	0.2263	0.0000	0.2263	0.6131	0.3869	0.1018
C9	1.0000	0.3869	0.1018	0.6131	0.1018	0.3869	0.3869	0.2263	1.0000	0.0000
C10	0.1018	0.0000	1.0000	1.0000	1.0000	0.2263	0.6131	0.0000	0.6131	0.2263
C11	0.1018	0.3869	0.2263	0.1018	0.2263	1.0000	0.0000	0.3869	0.0000	0.6131
C12	1.0000	0.1950	1.0000	0.5283	1.0000	0.1950	0.0000	0.8617	0.8617	0.3333
C13	0.8617	0.0877	0.0000	1.0000	0.1950	0.3333	1.0000	0.0877	0.1950	1.0000

Table 8. Information content of expert e_2.

	A1	A2	A3	A4	A5	A6	A7	A8	A9	A10
C1	1.0000	0.0313	0.5482	0.3649	0.1599	0.0313	0.7445	0.0000	0.5482	0.4006
C2	0.0877	1.0000	0.5283	0.1950	0.0000	0.5283	0.3333	0.0877	0.0000	0.5283
C3	0.5283	0.3333	0.8617	0.0000	0.3333	0.8617	0.5283	0.1950	0.5283	1.0000
C4	0.2263	0.0000	0.2263	0.6131	0.2263	0.6131	0.1018	0.1018	1.0000	0.2263
C5	0.0000	1.0000	0.1018	0.2263	0.1018	0.3869	1.0000	1.0000	0.1018	1.0000
C6	0.3869	0.2263	0.3869	0.0000	1.0000	0.1018	0.2263	0.0000	0.2263	0.1018
C7	0.6131	0.1018	0.2263	0.1018	0.3869	0.2263	1.0000	0.1018	0.1018	0.3869
C8	0.0877	0.5283	1.0000	0.3333	0.3333	0.0877	0.5283	0.1950	1.0000	0.0000
C9	0.1950	0.3333	0.0000	0.0877	1.0000	0.3333	0.0000	0.3333	0.3333	0.5283
C10	0.3869	0.0000	0.6131	1.0000	0.2263	0.2263	0.2263	0.6131	0.0000	1.0000
C11	0.6131	0.0000	0.2263	0.3869	0.6131	1.0000	0.2263	1.0000	0.3869	0.0000
C12	1.0000	0.1018	0.0000	1.0000	0.3869	0.1018	1.0000	0.2263	0.6131	0.1018
C13	1.0000	0.6131	1.0000	1.0000	1.0000	0.0000	0.0000	0.6131	0.3869	0.0000

Table 9. Information content of expert e_3.

	A1	A2	A3	A4	A5	A6	A7	A8	A9	A10
C1	0.5159	0.8267	0.1923	0.3483	1.0000	0.0000	0.4871	0.1923	0.3483	0.6659
C2	0.1018	1.0000	0.2263	0.0000	0.6131	0.3869	0.2263	0.1018	1.0000	0.3869
C3	0.0000	0.0877	0.0877	0.1950	1.0000	0.5283	0.8617	0.3333	0.0877	0.0000
C4	0.5283	1.0000	0.1950	0.0000	0.5283	0.1950	0.3333	0.5283	0.1950	0.0877
C5	0.1660	1.0000	0.6309	1.0000	0.0000	0.6309	0.1660	0.3691	0.1660	0.3691
C6	0.1950	0.1950	0.0000	0.3333	0.3333	0.1950	0.8617	0.0000	1.0000	0.0000
C7	0.0000	0.3869	1.0000	0.1018	1.0000	0.0000	0.2263	0.1018	0.3869	0.6131
C8	1.0000	0.0000	0.1660	0.6309	0.3691	0.1660	1.0000	0.0000	0.3691	0.6309
C9	0.3869	0.2263	0.6131	0.2263	0.1018	1.0000	0.0000	0.2263	0.6131	1.0000
C10	0.1950	0.8617	0.3333	0.0877	0.3333	0.5283	1.0000	0.3333	0.0000	1.0000
C11	1.0000	0.6131	0.0000	1.0000	0.2263	0.1018	0.0000	0.6131	0.3869	0.1018
C12	1.0000	0.3333	0.5283	1.0000	0.0877	0.0000	0.5283	0.8617	0.8617	0.5283
C13	0.8617	0.0000	1.0000	0.8617	0.8617	0.8617	0.0877	1.0000	0.0000	0.8617

Table 10. Information content of expert e_4.

	A1	A2	A3	A4	A5	A6	A7	A8	A9	A10
C1	0.7931	0.0000	0.3476	0.1919	0.0939	0.7604	0.1919	0.0700	0.8941	1.0000
C2	0.5283	0.8617	0.0000	0.3333	0.0877	0.8617	0.3333	0.8617	0.1950	1.0000
C3	0.2263	0.6131	1.0000	1.0000	0.6131	0.0000	0.1018	0.3869	0.1018	1.0000
C4	0.0000	0.1660	0.6309	0.1660	0.3691	0.0000	1.0000	0.3691	1.0000	0.3691
C5	0.1018	0.0000	1.0000	0.2263	1.0000	0.2263	1.0000	0.6131	0.0000	0.6131
C6	1.0000	1.0000	0.2630	1.0000	0.0000	1.0000	1.0000	0.5850	0.5850	0.0000
C7	0.3691	0.1660	0.6309	1.0000	0.3691	1.0000	0.0000	1.0000	0.1660	0.6309
C8	1.0000	0.0000	0.2263	0.1018	0.0000	0.1018	0.2263	0.1018	1.0000	0.2263
C9	0.0000	0.1950	1.0000	0.1950	0.0877	0.1950	1.0000	0.0000	0.3333	0.8617
C10	0.3869	0.6131	0.1018	0.0000	0.3869	0.6131	0.2263	0.3869	1.0000	0.1018
C11	1.0000	0.1950	0.5283	0.5283	0.5283	0.3333	0.0000	0.0877	0.5283	0.3333
C12	0.8617	0.3333	0.1950	0.0000	0.8617	0.8617	0.5283	0.0000	1.0000	0.5283
C13	0.5283	0.8617	0.0000	1.0000	0.3333	1.0000	0.0877	0.8617	0.3333	0.0877

Step 3. By solving model (6), the attribute weight vector with respect to different DMs can determined as:

$$\boldsymbol{w}^1 = \begin{pmatrix} 0.0788, 0.0858, 0.0718, 0.0902, 0.0721, 0.0823, 0.0681, \\ 0.0888, 0.0767, 0.0644, 0.0893, 0.0684, 0.0632 \end{pmatrix}^{\mathrm{T}},$$

$$\boldsymbol{w}^2 = \begin{pmatrix} 0.0749, 0.0814, 0.0788, 0.0880, 0.0589, 0.0986, 0.0955, \\ 0.0717, 0.0929, 0.0692, 0.0698, 0.0620, 0.0583 \end{pmatrix}^{\mathrm{T}},$$

$$w^3 = \begin{pmatrix} 0.0859, 0.0764, 0.0792, 0.0935, 0.0754, 0.0817, 0.0730, \\ 0.0718, 0.0757, 0.0724, 0.0701, 0.0750, 0.0701 \end{pmatrix}^{\mathrm{T}},$$

$$w^4 = \begin{pmatrix} 0.0723, 0.0750, 0.0696, 0.0772, 0.0664, 0.0694, 0.0726, \\ 0.0865, 0.0729, 0.0935, 0.1019, 0.0731, 0.0695 \end{pmatrix}^{\mathrm{T}}.$$

Step 4. By solving model (13), the weights of DMs can be derived and shown as follows:

$$\lambda_1 = 0.3868,\ \ \lambda_2 = 0.3558,\ \ \lambda_3 = 0.2207, \text{ and } \ \lambda_4 = 0.0366.$$

Step 5. According to Eq. (8), the total information content of each alternative can be calculated, the results are shown in Table 11.

Table 11. Total information content (TIC) of each alternative.

Alternative	e_1	e_2	e_3	e_4	**TIC**
a_1	0.4489	0.4546	0.4505	0.5400	0.4546
a_2	0.3591	0.3118	0.5148	0.3773	0.3773
a_3	0.4548	0.4259	0.3716	0.4440	0.4257
a_4	0.4735	0.3688	0.4296	0.4248	0.4247
a_5	0.3576	0.4572	0.5027	0.3604	0.4251
a_6	0.3560	0.3506	0.3464	0.5235	0.3581
a_7	0.4653	0.4415	0.4479	0.4173	0.4512
a_8	0.4113	0.3069	0.3542	0.3929	0.3609
a_9	0.3160	0.3990	0.4185	0.5706	0.3774
a_{10}	0.4254	0.4022	0.4680	0.5025	0.4293

Step 6. The alternatives can be ranked according to their total information content. The smaller the total information content, the better the alternative. It is easy to conclude from Tab. 3 that the ranking order of alternatives is $a_6 \succ a_8 \succ a_2 \succ a_9 \succ a_4 \succ a_5 \succ a_3 \succ a_{10} \succ a_7 \succ a_1$.

5.3 Comparative analysis

To show advantage of the proposed axiomatic design-based HMAGDM, it is compared with Yu's et al. [7] and Mao's et al. [30] methods.

5.3.1 Compare with Yu's et al. [7] method

Yu et al. [7] proposed a distance-based MAGDM method. According to Yu's et al. [7] method, the ranking order o f alternatives is $a_4 \succ a_3 \succ a_9 \succ a_7 \succ a_8 \succ a_{10} \succ a_2 \succ a_1 \succ a_6 \succ a_5$. It is easy to see that the ranking orders derived by the proposed method and Yu's et al. [7] method are quite different. The possible reasons are discussed as follows:

(i) Yu et al. [7] converted real numbers and LTs into interval-valued intuitionistic fuzzy numbers (IVIFNs), which results in information loss. Hence, the converted IVIFNs are hard to reflect DMs' real intention.

(ii) In Yu's et al. [7] method, there are a lot of parameters that need to be given, such as the optimistic coefficients. Obviously, different coefficients will result in different sorting results.

(iii) In Yu's et al. [7] method, the weights of DMs are given subjectively. Clearly, different weights of DMs would yield different ranking results.

5.3.2 Compare with Mao's et al. [30] method

Mao et al. [30] proposed a cloud model-based HMAGDM method. In Mao's et al. [30] method, the LTs are coded int o cloud, where expectation Ex, entropy En and hyper entropy He are three essential parameters. Assume that the values of

En and He are set as 1 and 0.0667, respectively. Then, according to Mao's et al. [30] method, the seven LTs listed in Table 1 can be coded into seven clouds, shown as Figure 2.

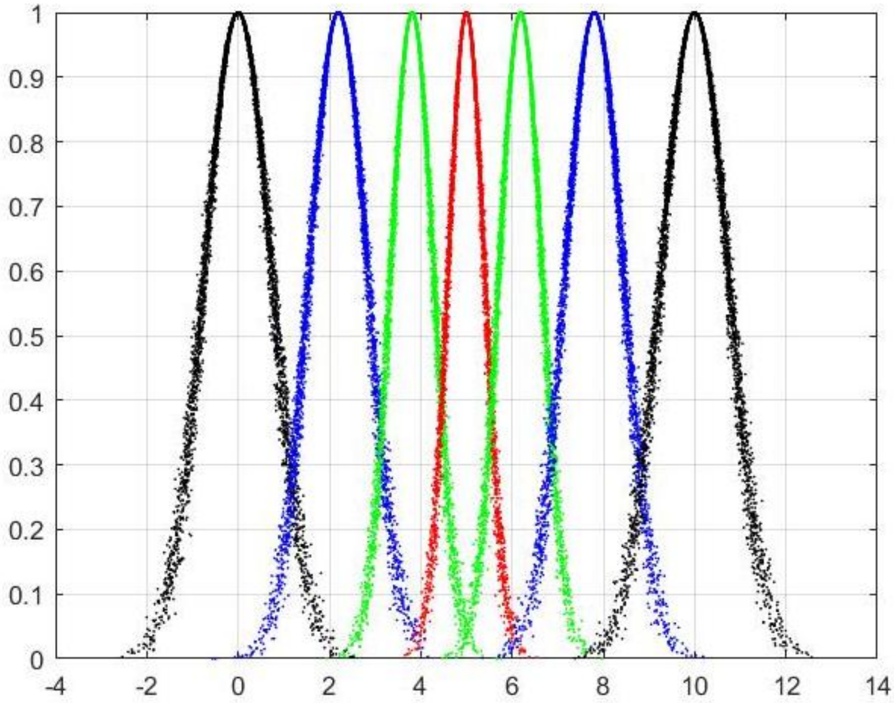

Figure 2. Schematic diagram of cloud model corresponding to linguistic variables.

Suppose that the weights of attributes (DMs) are equal. Then, the evaluation values (in the form of clouds) are obtained, shown in Table 12.

Table 12. Final evaluation value of each alternative.

Alternative	Evaluation value		
	Expectation Ex	Entropy En	Hyper entropy He
a_1	5.1957	0.5799	0.0387
a_2	5.5792	0.5754	0.0384
a_3	5.3709	0.5821	0.0388
a_4	5.3957	0.5832	0.0389
a_5	5.2452	0.5729	0.0382
a_6	5.4998	0.5696	0.0380
a_7	5.3583	0.5823	0.0388
a_8	5.5919	0.5775	0.0385
a_9	5.4167	0.5797	0.0387
a_{10}	5.2294	0.5832	0.0389

Based on Table 12, it can be observed that the corresponding cloud models for the ten alternatives are depicted in Figure 3.

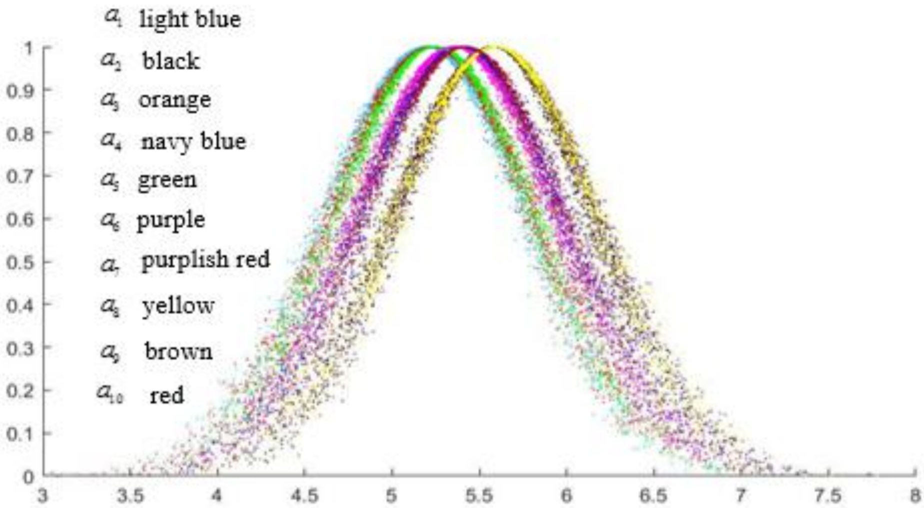

Figure 3. Cloud model corresponding to the overall evaluation value of alternative

Although the transformation of LTs into cloud model can express the uncertainty of LTs, the overall evaluation values of some alternatives are overlapped and hard to discriminated. From Fig. 2, it is easy to discover that ten alternatives can be roughly classified into three groups. There exists little differentiation within each group.

Both attribute weights and expert weights are considered to be equal. Reference [30] introduces the concept of Cloud Almost Stochastic Dominance (CASD) and constructs a three-objective optimization model from the perspectives of deviation maximization, correlation coefficient, and information entropy to solve for the attribute weights. By applying Equation (58) from Reference [30], the Cloud Almost Stochastic Dominance (CASD) degrees for each scheme under various attributes are aggregated with weights, resulting in the total CASD degrees for each scheme as follows:

$$q_1 = 0.4831,\ q_2 = 0.5604,\ q_3 = 0.4889,\ q_4 = 0.5414,\ q_5 = 0.4751,$$

$$q_6 = 0.4885,\ q_7 = 0.4803,\ q_8 = 0.5500,\ q_9 = 0.4981,\ q_{10} = 0.4341.$$

According to Ref. [30], the ranking order of alternatives is

$$a_2 \succ a_8 \succ a_4 \succ a_9 \succ a_3 \succ a_6 \succ a_1 \succ a_7 \succ a_5 \succ a_{10}.$$

In order to facilitate comparison, the total CASD (cloud almost stochastic dominance) of each alternative and the three digital features of each alternative are normalized. The normalization results are shown in Figure 4.

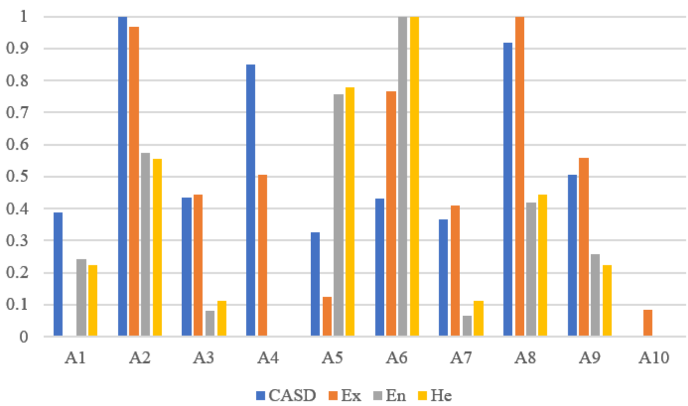

Figure 4. CASD degrees, Ex, En, He for 10 alternatives

Through comparison, it is found that the sorting results are unreasonable according to the total CASD, such as the sorting of a_4 and a_6. According to the total CASD, a_4 ranks the third and a_6 ranks the sixth. However, it is obvious from Fig. 4.4 that the values of the three digital features of a_6 are obviously better than that of a_4, but their ranking lags behind that of a_3. The reason for this problem is that the total CASD degree considers the weights of DMs, which are determined by the management parameters.

6. Conclusions

This paper analyzes the HMAGDM problem with real-valued attribute values and linguistic terms, where the attribute weight information and expert weight are completely unknown. A group decision making model based on axiomatic design is proposed for this problem. The model tries to use the original information given by experts as much as possible and builds a maximum deviation model objectively to determine the attribute weights based on the information volume. At the same time, it fully considers the differences among experts and proposes a method of determining expert weights based on maximizing the group consensus. Finally, the model ranks the solutions by integrating the information content. The main conclusions are as follows：

(1) The weights of DMs and attributes greatly affect the ranking results of MADM. For the same problem, different weights would lead to different ranking results. It is more reasonable and scientific to objectively solve attribute weights and expert weights through relevant mathematical models.

(2) There are many methods to quantify LTs. The proposed axiomatic design-based method can well express the uncertainty of LTs and have good discrimination ability.

(3) Most of the existing methods need to normalize the evaluation information, which surely results in serious information loss. The method proposed in this paper directly uses the original information and does not carry out standardized processing. It retains the original information to the greatest extent.

Currently, there are relatively rich research results on multi-attribute group decision making and preference relationship group decision-making both at home and abroad. However, there are not many decision-making methods that consider problems from the perspective of individual decision-makers and fully respect the decision makers' intentions, especially the research on heterogeneous preference relationships is relatively less. This may become a research direction in the future.

Ethical Approval

This article does not contain any studies with human participants or animals performed by any of the authors.

Conflict of Interest

The authors declare that they have no conflict of interest.

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