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Sinusoidal Curve Fitting and Moving Horizon Estimation Based Adaptive Unscented Kalman Filter for Geomagnetic Based Roll Attitude Estimation

Ting-ting Yin[1,2,*], Rui-sheng Sun[2], You-long Wu[1]

[1]*School of Intelligent Science and Control Engineering, Jinling Institute of Technology, Nanjing 211169, China*

[2]*School of Energy and Power Engineering, Nanjing University of Science and Technology, Nanjing 210094, China*

**Corresponding Author.*

Abstract:

The classical Unscented Kalman filter (UKF) algorithm will be deteriorated or even divergent while the prior information of the noises statistical characteristics of a nonlinear dynamic system is unknown or inaccurate. In order to avoid these constraints of classical UKF applied in geomagnetic based projectile roll attitude estimation, t a new adaptive UKF strategy by introducing in the sinusoidal curve fitting (SCF) and moving horizon estimation (MHE) is proposed in this paper. This strategy establishes an estimation principle of systematic and observing noise statistics with MHE and SCF combination. Based on the estimated noise statistics, the adaptive UKF with the estimated statistics as input parameters is designed. Consequently, a new optimized technique for geomagnetic based projectile roll attitude estimation is developed. The proposed adaptive UKF solution is able to realize the online computation of the noise statistics and enhance the stability and adaptation to variable application conditions of traditional filters. The advantages of the proposed adaptive UKF strategy are clearly illustrated by projectile roll attitude estimation results in the trajectory simulations.

Keywords: Unscented Kalman filter, noise statistic estimation, sinusoidal curve fitting (SCF), nonlinear moving horizon estimator (MHE), projectile roll attitude estimation.

INTRODUCTION

As for the most critical actuator roll position system of the trajectory correction projectile, the roll attitude control quality largely depends on the attitude estimation accuracy of the projectile body, where the strong nonlinearity and high dynamics will lead to large errors and severe distortions in the solution results[1]. The output weak analog signals of the strap-down geomagnetic sensor, which is the only but effective option to realize roll attitude positioning, can be easily interfered or distorted[2,3]. The noised geomagnetic signal can be observed simultaneously in the time- frequency domain where the timer captured frequency value and AD sampling amplitude data are respectively corresponding to the noisy roll velocity and position characteristics, while the much more noised positioning parameters urgently need to be optimized[4].

Since the roll speed decreases from 500r/s to 150r/s in the whole trajectory, where the single roll position estimation cycle lasts 6.7ms at most, the nonlinearity and variance of this single roll cycle is limited. Meanwhile, the optimized estimation based on the high frequency AD acquisition requires more simplified and effective method. Various filtering approaches, including slip averaging, moving horizon estimators (MHE) and Kalman filters have been developed. Slip averaging strategy, leading to estimating and controlling lagging errors, is not appropriate for nonlinear dynamic systems. MHE strategy, adopting iterative solution, containing repeated integration and comparison operations, cannot complete optimization calculation during AD acquisition intervals (less than 0.1ms)[5,6]. Nonlinear Kalman filters, including extended Kalman filters (EKF), unscented Kalman filters (UKF), and so on, have already been widely used in optimization applications of observed sequences. As for EKF algorithms, a first-order Taylor expansion is performed for the nonlinear system model, where the nonlinearities and dynamics should be limited in acceptable range, not appropriate for the projectile roll attitude optimization[7]. In consideration of the UKF methods, the advantages of simplified steps, improved precision and better convergence have laid solid application foundations [8,9]. However, the own limitations, such as the reliability of accurate prior statistical information of system noises, also cannot be ignored[10-12]. Soken H E and Wang Q T respectively introduced in scaling factor and adaptive factor to design robust and adaptive robust UKF algorithms, effectively weakening the filtering accuracy influenced from system model uncertainty, but the factors selected all by experiences cannot fundamentally solve the UKF shortages[13,14]. Afterwards, the online noise statistical estimator is designed and applied in UKF frames to improve the adaptive performance for random noises and disturbances, meanwhile, the computation amount rapidly increased to slow down real-time response, thus not appropriate for the high spin projectile circumstances[15,16].

In order to realize UKF optimization steps, the input states and noise statistical parameters should be accurately provided. As for UKF input state parameters, NMHE is a practical solution of the problem of online optimization or correction of nonlinear system model, using the data in a fixed length window, reducing the dimensions and computational load of optimization problems, providing the real-time optimization of projectile roll rate for UKF inputs. As for UKF input noise statistical parameters, the amplitude and frequency characteristics of discrete geomagnetic sequences are analyzed in consideration of the on-board geomagnetic sensors.

An adaptive UKF algorithm based on sinusoidal curve fitting and moving horizon estimation is proposed. Firstly, the measuring scheme as well as the classical UKF method is demonstrated; secondly, the SCF and MHE are respectively introduced to realize accurate state initial parameters evaluation and noise statistic estimation; finally, the SCF and MHE based adaptive UKF is established. The superiority of the proposed optimization algorithm is evaluated in comparison of classical UKF by the experimental data.

PROBLEM FORMULATION

In order to clearly illustrate the necessity and importance of the proposal of the SCF and MHE based adaptive UKF strategy, the problem formulation based on the nonlinear dynamic model as well as the classical UKF is introduced first.

Engineering Problem Formulation

The denoised geomagnetic sequence after wavelet reconstruction, presenting much better stability in the time domain, is a reliable data source for the projectile roll attitude solution.

Engineering solutions of roll attitude angle based on constant one-cycle roll rate hypothesis and special point information(such as zero crossing points) in geomagnetic measurement is introduced, as shown in Figure 1. Firstly, a zero-crossing comparison circuit is used to convert the sinusoidal geomagnetic output into continuous square waves with the synchronous frequency and phase information. The update of roll velocity measurement can be triggered at each rising edge of the initial phase Φ_0, where the rising edges are captured by the on-board processor, avoiding the time accumulation errors in conventional numerical solutions. Secondly, the roll attitude value of the projectile is calculated according to the dynamic projectile roll attitude model and the amplitude of the denoised geomagnetic sequence.

The measurement scheme for projectile roll attitude estimation based on the above engineering solution is proposed, as shown in Figure 1. $t_{P0}\left(k_P\right)$, $\omega_{P0}\left(k_P\right), \phi_0\left(k_P\right)$ are respectively the time, roll velocity and initial roll attitude at the rising edge of the k_Pth roll cycle, and $t_P\left(k_P, i_P\right)$, $\omega_P\left(k_P, i_P\right), \phi\left(k_P, i_P\right)$ represent the measured time, roll velocity and attitude of current sampling point of the wavelet denoised geomagnetic sequence, where the counted value of projectile roll cycle and the AD sampling points in one roll cycle are k_P and i_P.

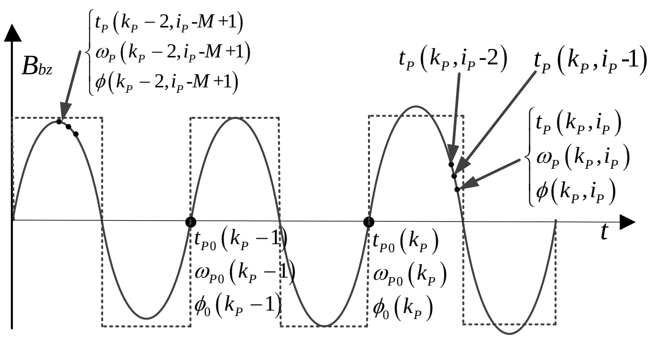


Figure 1. The measuring sequence of projectile roll attitude.

According to the detecting scheme in Figure 1, the sampling rate of the measurement data of projectile roll angle velocity, which is obtained based on timer edge capture method, denoting as t_{P0}, ω_{P0}, φ_0, is the projectile roll angle velocity, showing obvious time-varying nonlinearity. As for the roll attitude detecting scheme, where the results are denoted as t_P, ω_P, φ, the solution frequency, based on the AD sampling and denoising geomagnetic sequence, is a normal value settled by processor. Therefore, the two sets of solved data cannot be optimized in one state equation due to different trigger timings and updating rates. In addition, the directly timer captured velocity data and the indirectly model solved attitude data unavoidably contain various errors and disturbances, where the simple linear or nonlinear interpolation method may introduces in more errors in the contrary.

In conclusion, with the simplified engineering solution and the initial attitude parameters from zero-crossing points, the roll attitude optimization should be accomplished during AD sampling intervals in one projectile roll cycle, where the captured roll velocity value is applied.

Classical UKF for Roll Attitude Estimation

In order to clearly demonstrate the enhancement of the established adaptive UKF estimator on the basis of MHE and SCF, the classical UKF is reviewed at first.

Considering the roll attitude estimation method, the captured roll velocity value is substituted in the dynamic solution model as shown in the discrete equations below:

$$\begin{cases} x_{k+1} = f(x_k, u_k) + w_k \\ y_k = h(x_k) + v_k \end{cases} \tag{1}$$

Where 1(a) refers to the dynamic procedure of the projectile body and 1(b) refers to the observing equations where *f* and *h* are both continuous functions. $x_k \in R^n$ represents the state vector while the original state x_0 is not given. $u_k \in R^n$ means the input parameters from the on-line control algorithm, meanwhile, $w_k \in R^m$ means the unexpected external disturbances. $y_k \in R^p$ represents the observation vectors and $v_k \in R^p$ represents the measurement noises and errors. The external interferences and observation noises have nothing to do with the state vectors, and are assumed to be zero mean Gaussian distributed, while noise variables and dynamic interference are bounded in a relatively small range.

The classical UKF based attitude optimization procedure is summarized below:

Step 1: Preparation. In this schedule, the prior state estimation $\hat{X}(0|0)$ and covariance $\hat{P}(0|0)$, covariance R^w and R^v for systematic errors and observing noises should be pre-estimated as inputs. Model parameters such as $\kappa \ge 0, \alpha \ge 1, \beta \ge 0$, dimension numbers *n*, as well as the scaling value $\lambda = \alpha^2 * (n+\kappa) - n$ are all initialized. Meanwhile, the weights and respective weighted sample points, noted as W_i $X_i(k-1|k-1)$ are prepared.

$$\begin{aligned} &X_0(k-1|k-1) = \hat{X}(k-1|k-1) && i=0 \\ &X_i(k-1|k-1) = \hat{X}(k-1|k-1) + \sqrt{(n+\lambda)\hat{P}(k-1|k-1)} && 1 \le i \le n \\ &X_i(k-1|k-1) = \hat{X}(k-1|k-1) - \sqrt{(n+\lambda)\hat{P}(k-1|k-1)} && n < i \le 2n \end{aligned} \tag{2}$$

$$\begin{aligned} &W_0^{(m)} = \frac{\lambda}{n+\lambda} \\ &W_0^{(c)} = \frac{\lambda}{n+\lambda} + (1-\alpha^2+\beta) \\ &W_i^{(m)} = W_i^{(c)} = \frac{1}{2(n+\lambda)} \quad i = 1,\ldots,2n \end{aligned} \tag{3}$$

Step 2: Prediction of state $\hat{X}(k|k-1)$, measurement $\hat{Y}(k|k-1)$ and covariance $\hat{P}(k|k-1)$.

$$\begin{gathered} X_i(k|k-1) = f(X_i(k-1|k-1), u(k-1)) \\ X_i(k|k-1) = f(X_i(k-1|k-1), u(k-1)) \quad i = 0,\ldots,2n \\ \hat{X}(k|k-1) = \sum_{i=0}^{2n} W_i^{(m)} X_i(k|k-1) \\ Y_i(k|k-1) = h(X_i(k-1|k-1)) \\ \hat{Y}(k|k-1) = \sum_{i=0}^{2n} W_i^{(m)} Y_i(k|k-1) \\ \hat{P}(k|k-1) = \sum_{i=0}^{2n} W_i^{(c)} \left([X_i(k|k-1) - \hat{X}(k|k-1)] \cdot [X_i(k|k-1) - \hat{X}(k|k-1)]^T \right) + R^w \end{gathered} \tag{4}$$

Step 3: Updating the auto-correlated error $\hat{P}yy$ and cross-correlated error $\hat{P}xy$.

$$\begin{aligned}\hat{P}yy &= \sum_{i=0}^{2n} W_i^{(c)}\left([Y_i(k|k-1)-\hat{Y}(k|k-1)]\cdot[Y_i(k|k-1)-\hat{Y}(k|k-1)]^T\right)+R^V \\ \hat{P}xy &= \sum_{i=0}^{2n} W_i^{(c)}\left([X_i(k|k-1)-\hat{X}(k|k-1)]\cdot[Y_i(k|k-1)-\hat{Y}(k|k-1)]^T\right)\end{aligned} \tag{5}$$

Step 4: Estimation of the optimized state vector $\hat{X}(k|k)$ and covariance $\hat{P}(k|k)$.

$$\begin{aligned}&\hat{K}(k)=\hat{P}xy/\hat{P}yy \\ &\hat{X}(k|k)=\hat{X}(k|k-1)+\hat{K}(k)(Y(k)-\hat{Y}(k|k-1)) \\ &\hat{P}(k|k)=\hat{P}(k|k-1)+\hat{K}(k)\hat{P}yy\left(\hat{K}(k)\right)^T\end{aligned} \tag{6}$$

Step 5: Repetition of step 2-4 where $\hat{X}(k-1|k-1)$ and $\hat{P}(k-1|k-1)$ are replaced with the new gained estimation $\hat{X}(k|k)$ and $\hat{P}(k|k)$.

It can be seen from the classical UKF that if the noise statistic R^w and R^v are not accurate, the prior state covariance $\hat{P}(k|k)$ will be diverged, leading to the diverged solved state vectors. In addition, as the important input state parameters, the accuracy of velocity value as well as the captured timing information also makes great sense for the resolution and reliability of classical UKF estimators.

As for the engineering projectile roll attitude estimation condition, the disturbances and noises of the dynamic projectile roll model as well as the measurement solution has strong nonlinear and time-varying characteristics. In addition, the detected roll velocity information contains unavoidable errors and disturbances. From the above analysis, the solution based on the traditional UKF method is very likely to be burred or even non-convergence, not applicable for time-varying nonlinear systems.

PROPOSED NEW ADAPTIVE UKF BASED ON MHE AND SCF

Accurate State Initial Parameters Evaluation Based on MHE

In order to realize accurate estimation of roll attitude, the directly detected roll velocity value should be optimized. Moving Horizon Estimation is applied to realize real-time velocity optimization to provide accurate state initial parameters for UKF.

The dynamic equation and observation equation of projectile roll angle rate based on rolling time domain estimation can be simply expressed as:

$$\begin{cases}\omega_{P0}(k_P+1)=f_P(\omega_{P0}(k_P),u(k_P))+w_P(k_P) & (a)\\ y_{\omega_{P0}}(k_P)=h_P(\omega_{P0}(k_P))+v_P(k_P) & (b)\end{cases} \tag{7}$$

Where $w_P(k_P)$, $v_P(k_P)$, $u(k_P)$ are respectively the system disturbance/observation noise and control input parameters; f_P and h_P are continuous functions; the disturbance and noise are assumed to be bounded and obey zero mean Gaussian distribution, that is, $w'_P \in W_P, v_P \in S_P$.

In the rolling time domain window [$t_{P0}(k_P\text{-}N+1)$, $t_{P0}(k_P)$] with width N, the system parameter set $I_P(k_P)$ is expressed as:

$$I_P(k) \square\ col(y_{\omega_{P0}}(k_P-N+1),\ldots,y_{\omega_{P0}}(k_P),u(k_P-N+1),\ldots,u(k_P-1)) \tag{8}$$

As for $\omega_{P\min} \le \hat{\omega}_{P0} \le \omega_{P\min}$, based on MHE method, the formula for solving the estimated value of state optimization $\hat{\omega}_{P0}(k_P\text{-}N+1)$ at $t_{P0}(k_P\text{-}N+1)$ is:

$$\hat{\omega}_{P0}(k_P-N+1) \in \arg\min J_\omega(\hat{\omega}_{P0}(k_P-N+1),\bar{\omega}_{P0}(k_P-N+1),I_P(k_P)) \tag{9}$$

Where $\bar{\omega}_{P0}(k_P-N+1)=f_P(\hat{\omega}_{P0}(k_P-N),u(k_P-N))$ is the initial estimated value calculated based on the optimized value of the previous moment and the rolling motion equation of the projectile; $\omega_{P\min}$ and $\omega_{P\max}$ is the maximum and minimum values of the roll rate of the projectile determined according to the previous flight tests.

However, the optimization object of traditional rolling time domain estimation is the initial point of time domain window. If the rotational speed is estimated at N points from the current rotational speed, the hysteresis caused by this method is $t_{P0}(k_P)$-$t_{P0}(k_P$-N+1), which cannot meet the real-time requirements of the body spin speed. In duck rudder roll control and even trajectory correction system, the real-time and accuracy of projectile roll rate are indispensable. Therefore, it is necessary to improve the traditional rolling time domain estimation method, and satisfy the online and accuracy demands of the estimation of projectile roll rate.

The inverse function of continuous derivable function f_P can be derived:

$$\begin{cases}\omega_{P0}(k_P)=f_P^{-1}(\omega_{P0}(k_P+1),u(k_P))-w'_P(k_P) & (a)\\ y_{\omega_{P0}}(k_P)=h_P(\omega_{P0}(k_P))+v_P(k_P) & (b)\end{cases} \tag{10}$$

Under the condition of keeping the time domain window, i.e. parameter set, unchanged, the real-time optimized speed value can be solved by the following formula:

$$\hat{\omega}_{P0}(k_P)\in\arg\min J_\omega(\hat{\omega}_{P0}(k_P),\bar{\omega}_{P0}(k_P),I_P(k_P))\quad \omega_{P\min}\le\hat{\omega}_{P0}\le\omega_{P\min} \tag{11}$$

Where:

$$\bar{\omega}_{P0}(k_P)=f(\hat{\omega}_{P0}(k_P-1),u(k_P-1)) \tag{12}$$

$$J_\omega=\mu\left\|\hat{\omega}_{P0}(k_P)-\bar{\omega}_{P0}(k_P)\right\|^2+\sum_{i_0=k_P-N+1}^{k_P}\left\|y_{\omega_{P0}}(i_0)-h(\tilde{\omega}_{P0}(i_0))\right\|^2 \tag{13}$$

And:

$$\tilde{\omega}_{P0}(i_0)=\begin{cases}f_P^{-1}(\bar{\omega}_{P0}(i_0+1),u(i_0)) & i_0\ne k_P\\ \hat{\omega}_{P0}(k_P) & i_0=k_P\end{cases} \tag{14}$$

In can be concluded that, under the condition of keeping the rolling time domain window unchanged, the improved rolling time domain estimation method shifts the optimization target from the initial point of the window to updating the time data points in real time, and meets the requirements of accuracy and timeliness.

SCF Based Noise Statistic Estimation

In order to guarantee the stability of the UKF optimization strategy, the SCF method has been introduced in to obtain on-line noise statistics of the dynamic and observation equations.

Combined with the approximate sinusoidal characteristics of geomagnetic sensor output sequence, an adaptive UKF noise statistical estimation algorithm based on rolling time domain and sinusoidal fitting method is proposed in this work. The estimation principle of this method is as follows: firstly, the geomagnetic acquisition sequence is intercepted according to the sampling rate and periodic characteristics of the system; then, the estimation model of system measurement noise statistics is constructed according to the sinusoidal fitting algorithm in the time domain window; and the statistical estimation model of system noise introduced by the assumption of constant speed in a single cycle is constructed in a single cycle.

The velocity of projectile roll angle decreases with time, that is, the period time of geomagnetic sequence increases gradually. Generally, the A/D sampling rate is fixed, and if the time domain window interception method with fixed points is adopted, the non-positive period truncation of geomagnetic signal will inevitably result. Based on the nonlinear characteristics of geomagnetic sequence and projectile rolling period, this section proposes a rolling time domain window interception method based on the previous complete projectile rolling period, as shown in Figure 2.

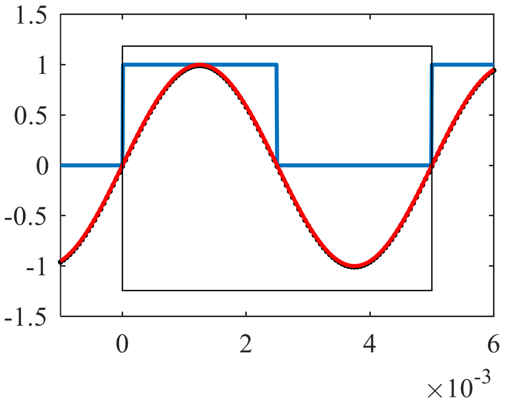


Figure 1. The rolling time domain window with fixed width.

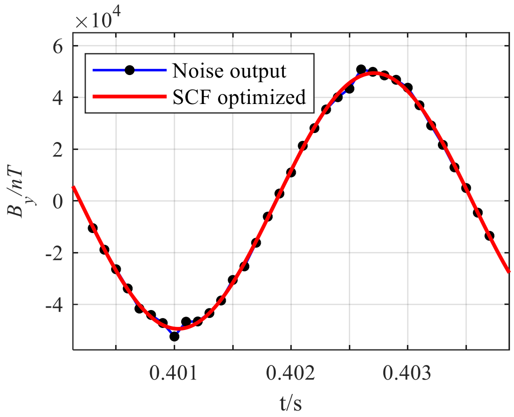


Figure 2. The de-noised geomagnetic curve by SCF method.

Taking the rising zero-crossing time of the geomagnetic sinusoidal sequence in a complete projectile rolling period as a window, as shown in Figure 2, the geomagnetic sequence in the range of [$t_{P0}(k_P$-1) $t_{P0}(k_P)$] is intercepted and recorded as $Y_P=\left[y_P(1),\dots y_P(i_P),\dots y_P(N_P(k_P-1))\right]$. Different from the rolling time domain window with fixed width, the rolling time domain window width based on the rolling period of projectile, denoting as $N_P(k_P-1)$, is linearly related to the rolling period of projectile, which has nonlinear time-varying characteristics, and the data points in the window are in the complete rolling period.

At the beginning and the end of the rolling time domain window, the rolling angle rates obtained by the rolling time domain estimation method are respectively $\hat{\omega}_{p0}(k-1)$ and $\hat{\omega}_{p0}(k)$, and the variation of the rolling angle rate of the projectile is linearly approximated in a single period. At the same time, the geomagnetic sequence after wavelet de-noising is intercepted during the specified rolling window in the time domain, and the amplitude $A_P(k_P$-1) of this sequence is obtained by sinusoidal curve fitting (SCF), and the fitting results of the rolling phase of the projectile $\phi_1(i_P)$ and the geomagnetic sequence $y_{P1}(i_P)$, shown in Figure 3, are obtained as follows:

$$\begin{cases} y_{P1}=A_P\left(k_P-1\right)\sin(\phi_1) \\ \phi_1=\hat{\omega}_{p0}(k_P-1)(t_P(i_P)-t_{P0}\left(k_P-1\right))+\dfrac{1}{2}\dfrac{\hat{\omega}_{p0}(k_P)-\hat{\omega}_{p0}(k_P-1)}{t_{P0}\left(k_P\right)-t_{P0}\left(k_P-1\right)}(t_P(i_P)-t_{P0}\left(k_P-1\right))^2 \end{cases} \tag{15}$$

Then the statistical estimation model of system geomagnetic measurement noise $R^q(k_P-1)$ can be expressed as:

$$R^q(k_P-1)=\frac{\sum_{1}^{N_P(k_P-1)}(y_P-y_{P1})^2}{N_P(k_P-1)} \tag{16}$$

For the purpose of reducing the complexity of the algorithm when calculating the rolling phase of the projectile in a single rolling period, a rolling angle estimation model based on UKF is constructed by introducing the rolling angle rate obtained by rolling time domain estimation. Therefore, in the intercepted rolling time domain window, the phase difference introduced by the assumption of constant speed in one cycle is the main source of system model noise. The phase fitting result based on the assumption of constant speed can be expressed as:

$$\phi_2(i_P)=\frac{1}{2}(\hat{\omega}_{p0}(k_P)+\hat{\omega}_{p0}(k_P-1))(t_P(i_P)-t_{P0}\left(k_P-1\right)) \tag{17}$$

The noise statistical estimation model of system phase model is constructed as:

$$R^p(k_p-1)=\frac{\sum_{1}^{N_P(k_P-1)}(\phi_2-\phi_1)^2}{N_P(k_P-1)} \tag{18}$$

In each rolling cycle, the noise statistical parameters in UKF model are estimated online in real time by combining the above formula. The model noise statistics and measurement noise statistics of time-varying system are substituted into the rolling angle estimation model based on UKF in the next rolling period.

SCF and MHE Based Adaptive UKF Estimation Strategy

Based on the above illustration of SCF and MHE technology, the proposed adaptive UKF can be realized by the described and realized by the following steps:

Step1: Define UKF parameters $\kappa\geq 0,\alpha\geq 1,\beta\geq 0$ to establish state weight equations based on UKF.

$$\begin{aligned}
&X_{Pj}\left(i_P-1\,|\,i_P-1\right)=\hat{X}\left(i_P-1\,|\,i_P-1\right) && j=0\\
&X_{Pj}\left(i_P-1\,|\,i_P-1\right)=\hat{X}\left(i_P-1\,|\,i_P-1\right)+\sqrt{(n+\lambda)\hat{P}\left(i_P-1\,|\,i_P-1\right)} && 1\leq j\leq n\\
&X_{Pj}\left(i_P-1\,|\,i_P-1\right)=\hat{X}\left(i_P-1\,|\,i_P-1\right)-\sqrt{(n+\lambda)\hat{P}\left(i_P-1\,|\,i_P-1\right)} && n<j\leq 2n
\end{aligned} \tag{19}$$

Step2: Prediction.

$$\begin{aligned}
&X_{Pj}\left(i_P\,|\,i_P-1\right)=f(X_{Pj}\left(i_P-1\,|\,i_P-1\right),u(k_P-1)) \quad \hat{X}_P\left(i_P\,|\,i_P-1\right)=\sum_{j=0}^{2n}W_j^{(m)}X_{Pj}\left(i_P\,|\,i_P-1\right)\\
&Y_{Pj}\left(i_P\,|\,i_P-1\right)=h(X_{Pj}\left(i_P-1\,|\,i_P-1\right)) \quad \hat{Y}_P\left(i_P\,|\,i_P-1\right)=\sum_{j=0}^{2n}W_j^{(m)}Y_{Pj}\left(i_P\,|\,i_P-1\right)\\
&\hat{P}\left(i_P\,|\,i_P-1\right)=\sum_{j=0}^{2n}W_j^{(c)}[X_{Pj}\left(i_P\,|\,i_P-1\right)-\hat{X}_P\left(i_P\,|\,i_P-1\right)][X_{Pj}\left(i_P\,|\,i_P-1\right)-\hat{X}_P\left(i_P\,|\,i_P-1\right)]^{\mathrm{T}}+R^p
\end{aligned} \tag{20}$$

Where:

$$W_0^{(m)}=\frac{\lambda}{n+\lambda},\quad W_0^{(c)}=\frac{\lambda}{n+\lambda}+(1-\alpha^2+\beta),\quad W_j^{(m)}=W_j^{(c)}=\frac{1}{2(n+\lambda)}\qquad j=1,\ldots,2n$$

Step3: Update.

$$\begin{aligned}
&\hat{P}yy(i_P)=\sum_{j=0}^{2n}W_j^{(c)}[Y_{Pj}\left(i_P\,|\,i_P-1\right)-\hat{Y}_P\left(i_P\,|\,i_P-1\right)][Y_{Pj}\left(i_P\,|\,i_P-1\right)-\hat{Y}_P\left(i_P\,|\,i_P-1\right)]^{\mathrm{T}}+R^q\\
&\hat{P}xy(i_P)=\sum_{j=0}^{2n}W_j^{(c)}[X_{Pj}\left(i_P\,|\,i_P-1\right)-\hat{X}_P\left(i_P\,|\,i_P-1\right)][Y_{Pj}\left(i_P\,|\,i_P-1\right)-\hat{Y}_P\left(i_P\,|\,i_P-1\right)]^{\mathrm{T}}\\
&\hat{K}_P(i_P)=\hat{P}xy(i_P)/\hat{P}yy(i_P)\\
&\hat{X}_P\left(i_P\,|\,i_P\right)=\hat{X}_P\left(i_P\,|\,i_P-1\right)+\hat{K}(i_P)(Y_P(i_P)-\hat{Y}_P\left(i_P\,|\,i_P-1\right))\\
&\hat{P}\left(i_P\,|\,i_P\right)=\hat{P}\left(i_P\,|\,i_P-1\right)+\hat{K}_P(i_P)\hat{P}yy\left(\hat{K}_P(i_P)\right)^{\mathrm{T}}
\end{aligned} \tag{21}$$

Step4: Repeat above steps for the following AD sampling data in one projectile roll cycle.

In summary, firstly, based on the projectile dynamic roll model, the real-time ballistic data and control parameters are input into the MHE model to optimize the projectile roll angular rate value as the initial states of the adaptive UKF estimation algorithm. Secondly, taking the timer trigger edge as the beginning and end of the rolling time window to intercept the geomagnetic sampling sequence, and then the SCF is conducted to estimate the respective noise statistics to provide the prior statistics parameters of the adaptive UKF estimation algorithm. Finally, the geomagnetic sequence in the rolling time window after wavelet denoising as well as the above items is substituted as the observation inputs of the adaptive UKF algorithm to obtain the optimal estimation of roll attitudes.

SIMULATION AND EXPERIMENTAL PERFORMANCE EVALUATIONS

A prototype roll attitude estimation system based on the proposed adaptive UKF algorithms has been implemented. Thus, the trajectory simulations as well as experiments where the grounded semi-physical simulation platform is applied have also been implemented to comprehensively evaluate the proposed SCF and MHE based adaptive UKF algorithm for precise roll attitude estimation of the nonlinear and high-dynamic projectile system, as shown in Figure 4. Comparative analysis of estimation precision of the proposed algorithm with the traditional filters is executed and illustrated in this section.

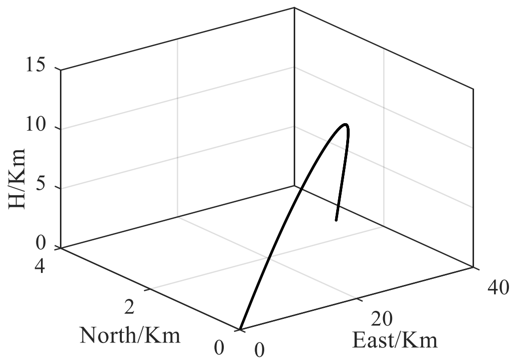
(a)

(b)

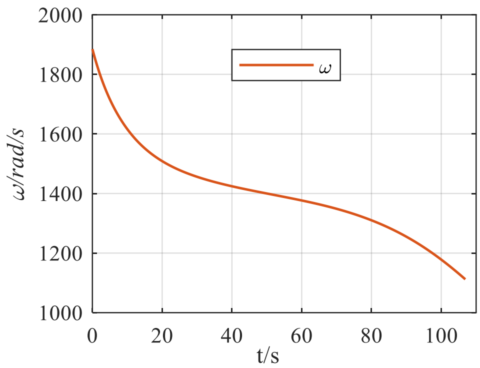
Figure 4. The prototypes for trajectory experiments.

SCF and MHE Based UKF Input Signal Optimization Performance Evaluation with Trajectory Simulations

So as to preliminarily confirm the superiority of the proposed algorithm, this paper utilizes the 155mm simulation ballistic curve to simulate and analyze the projectile's flight trajectory and the characteristic curve of the projectile's rolling angular rate as shown in Figure 5. It can be seen from the curve that the height and horizontal distance of the projectile's flight have reached nearly 20Km and 40Km, respectively, and there is a large change in the total amount of geomagnetism. It can be concluded from the figure that the rolling angular rate of the projectile presents a strong time-varying nonlinear characteristic, and the traditional optimization method is difficult to realize the high-precision observation of the rolling attitude of the projectile based on geomagnetism.



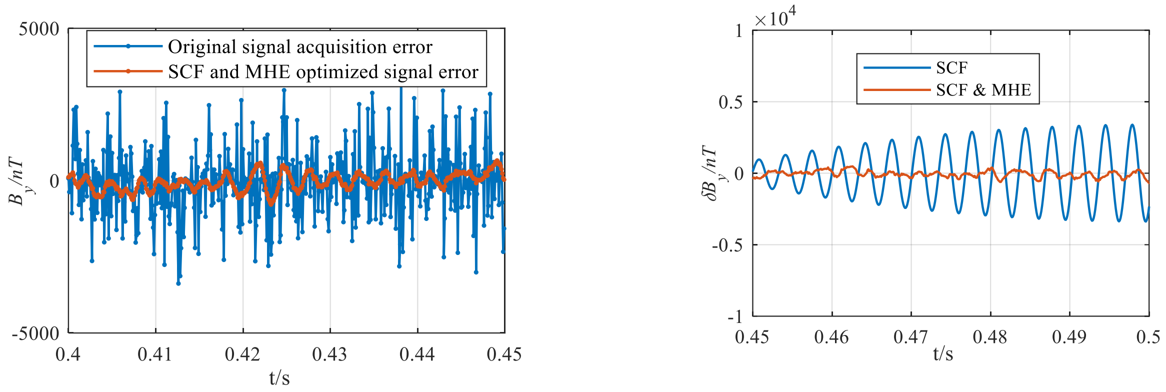
(a)



(b)

Figure 5. Trajectory characteristic curve of simulation models.

Taking the geomagnetic output curve in a short period as data source, the SCF, SCF&MHE methods are used to optimize the geomagnetic data, and the optimized real-time geomagnetic output signal and error distribution curves are shown in Figure 6. It can be seen that the combined method of SCF and MHE proposed in this paper can more effectively reduce the noise of the geomagnetic signals, so as to provide a more reliable data source for the subsequent optimization of the projectile roll attitude.



(a) (b)

Figure 6. Signal conditioning performance evaluation.

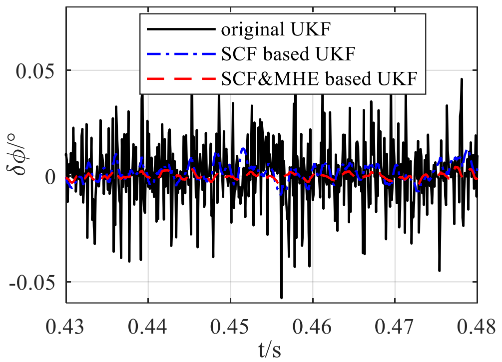


Figure 7. Roll attitude estimation performance under strong disturbed simulation conditions.

The simulation results of the optimization of the rolling attitude of the projectile are shown in Figure 7. From Figure 7, it can be intuitively seen that the method proposed in this paper can provide more accurate and reliable results of the rolling attitude of the projectile. Statistical analysis of the simulation results shows that the proposed UKF method based on SCF&MHE proposed in this paper can increase the RMS value from 0.16° of the UKF based on SCF method to 0.07°. The simulation results fully confirmed the feasibility and advantages of the strategy proposed in this paper.

SCF and MHE Based Adaptive UKF Attitude Estimation Performance Evaluation with Experimental Data

In order to more fully prove the superiority of the method proposed in the thesis, the performance of the method is analyzed and verified based on the test data of the artillery shot experiment. First of all, it can be seen from the curve comparison in the figure that the MHE method can effectively reduce the error distribution of the spin rate measurement of projectile, meanwhile, provides a more accurate state input value for the roll angle calculation based on the UKF, as shown in Figure 8.

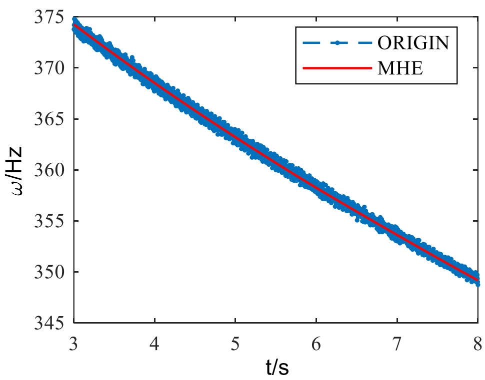


Figure 8. Roll velocity estimation performance.

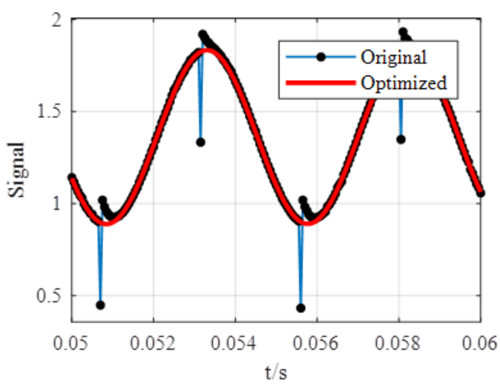


Figure 9. Geomagnetic signal conditioning performance 1.

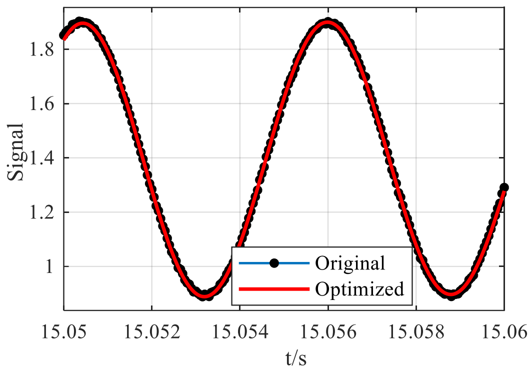


Figure 10. Geomagnetic signal conditioning performance 2.

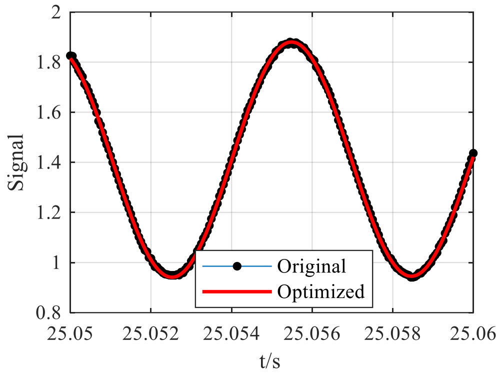


Figure 11. Geomagnetic signal conditioning performance 3.

Secondly, as the observation value of the UKF solution model, the SCF method can fully reduce the observation error data such as burrs, jitter, and jitter in the geomagnetic observation signal, and provides a smoother observation for the high-precision calculation of the rolling attitude of the projectile, as shown in Figure 9, Figure 10 and Figure 11.

Finally, while the result of the above optimization are applied as input parameters in the UKF optimization model, the optimized state statistics of the projectile roll posture solution after model optimization is shown in the Figure 12. From the perspective of a single roll cycle, the proposed UKF method based on MHE&SCF can reduce the original rolling calculation error of the projectile from 4e-4rad to 8e-7rad. From the long-term optimization results in Figure 13, this proposed UKF method based on MHE&SCF can more effectively suppress the nonlinear time-varying characteristics of projectile roll; thereby controlling the solution error within ±0.01rad, providing is a reliable input source of control parameters for subsequent projectile attitude and trajectory correction.

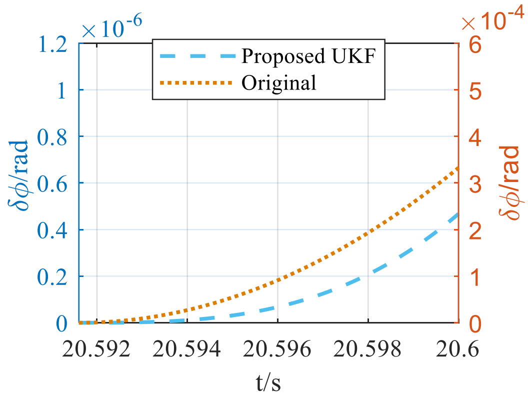


Figure 12. Roll attitude estimation performance in one cycle.

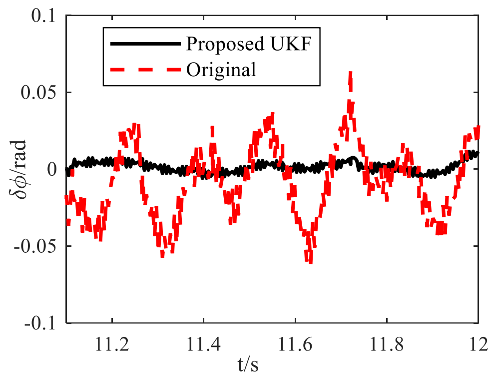


Figure 13. Roll attitude estimation performance comparison.

CONCLUSIONS

In this work, a novel adaptive Unscented Kalman filter (UKF) based on sinusoidal curve fitting (SCF) and moving horizon estimation (MHE) is implemented to precisely estimate the real-time roll attitude of the time-varying nonlinear projectile roll systems. The MHE and SCF methods respectively provide accurate state values, reliable noise statistics and observation sequences for the proposed UKF estimation model. It's obvious from simulation and experimental results and corresponding analysis that the proposed roll attitude optimization method can realize correct estimation and error reduction, thus, the feasibility and reliability of the proposed UKF optimization method based on MHE and SCF is verified. In our future research work, we will consider the necessary engineering simplifications in more practical applications with complicated dynamic and disturbed issues.

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CONFLICT OF INTEREST

The authors declare that they have no conflicts of interest.

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