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Application Research of Physical Model Based on Differential Equation Theory in Salvaging Dropped Items

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Abstract:
Incidents of items accidentally falling into water are common in daily life, especially during travel, where important items such as ID cards and mobile phones fall into water. This article uses the items falling into the West Lake and the Beijing-Hangzhou Grand Canal as examples. By utilizing the theory of differential equations advanced mathematics, a physical model is established. The model takes into account the impact of water flow speed, direction, wind speed, and other factors on the final point of the item. It provides a detailed simulation of the item's descent process in the water and analyzes the trajectory of the item's movement in the water. Python for numerical simulation, the recommended search radius for a mobile phone falling into the West Lake is obtained as 0.00042m, while a mobile phone falling into the Grand Canal, the recommended search radius is 0.18m, Similar analysis of ID search radius in West Lake and Grand Canal areas, These results provide theoretical support and scientific basis for actual retrieval work.

Keywords: differential equation, physical model, Python, dynamics equations, salvage area.

INTRODUCTION

The problem of object entering water, as a classic topic in fluid dynamics, holds significant theoretical and practical importance. This physical process involves complex-structure interactions, including the initial impact stage and subsequent underwater motion, and has garnered scholarly attention since the early 20th century. Since 2011, research related to the process of objects entering water has gradually enriched.

Li and Hu [1] studied the simulation of two-dimensional profiles entering water at constant speed based on potential flow theory and the boundary element method. Yang et al. [2] used an improved smoothed particle method to study the problem of objects water. Liang et al. [3] measured the position of the object's entry point into the water using a pure azimuth method and proposed a method to the position of the object's landing point in the water using an acoustic buoy system. Shi and Zhou [4] conducted research on the dynamics of objects exiting entering water, measuring the drag coefficient Cd of the object after it entered the water. There is also a rich history of experimental research on the problem of entering water May [5,6] conducted pioneering experimental studies on the entry of spheres and missiles into water, revealing the important impact of surface conditions and entry parameters on the of cavities. Recent experimental work by Duclau et al. [7] provides detailed insights into the dynamics of transient cavities, while Aristoff and Bush [8] specifically studied the behavior of small hydrophobic spheres entering water.

With the improvement of computing power, numerical simulation has become an increasingly important research tool in this field. Wang and Faltinsen [9] conducted a detailed numerical study on the cavity formation of wedge objects during high-speed water entry, while Moore et al. [10] addressed the challenging of three-dimensional oblique water entry. These studies have significantly enhanced the understanding of the complex fluid dynamics involved.

The review article by Tuscott et al. [11] systematically summarizes various aspects of water entry research, covering both classical results and modern developments. The work of Gekle and Gordillo [12] has further advanced the understanding of jet formation and cavity dynamics, demonstrating the ongoing development in this research area. The main research findings are reflected in the study of during the water entry process of objects, but there are few studies on the recovery of items based on these findings. In recent years, the research results of water and recovery problems based on physical models mainly include [13-17]. Although there has been extensive research on the recovery of objects from water, there are very studies that apply physical models to actual recovery operations, especially for common personal items such as mobile phones and ID cards.

In daily life, incidents of important items such as mobile phones and ID cards falling into the water are not uncommon, making the efficient and retrieval of such items an urgent issue to address. Currently, the main method for retrieval is manual blind search, which has an uncertain search range and is time-consuming Due to the complex underwater environment, low visibility, and limited tools available, the success rate of retrieval is low. Moreover, the lack of scientific basis and theoretical in existing methods leads to reliance on experience and intuition, without systematic analysis and optimization strategies. To significantly improve the efficiency and success rate of retrieval, it is urgent conduct in-depth research on the movement patterns of items falling into the water and to formulate scientific retrieval strategies based on this research.

The article provides theoretical support for retrieval operations by establishing a physical model to simulate the process of objects falling into a static body of water, such the West Lake. The study subjects include common objects and ID cards, analyzing their trajectories in the water. By simulating and analyzing the movement patterns of objects in water, and using the concept of differential equations, a physical model is constructed to detail the process of objects falling into a static body of water. Taking mobile and ID cards as examples, the model fully considers the significant impact of water flow speed and direction on the final landing point of the objects. It analyzes the movement of the objects in the lake using Python for numerical simulation. Based on the simulation results, retrieval strategies for objects lost in the water are determined, and an effective area is established, providing scientific retrieval evidence and theoretical support for actual retrieval operations. Meanwhile, the construction and application of the physical model based on differential equations is also specific application practice of the ideas and methods of advanced mathematics courses.

PHYSICAL MODEL INTRODUCTION

In order to provide theoretical support for salvage work, a physical model is constructed to simulate the falling process of objects in static waters in detail. The model mainly considers the various forces of objects in air and water, and uses dynamic equations and Python for numerical simulation.

Theoretical basis of the model:

(1) Newton's second Law[Error! Reference source not found.] : The motion of an object is affected by force, and any force is equal to mass times acceleration:

$$F = ma \tag{1}$$

(2) Air resistance and water resistance: According to the theory of fluid dynamics, objects will experience resistance when moving in air and water, which is usually related to the speed of the object and the density of the fluid. Among them:

The formula of air resistance[Error! Reference source not found.]:

$$F_{drag,air} = \frac{1}{2} c_d \rho_{air} A v^2 \tag{2}$$

The formula of water resistance [Error! Reference source not found.]:

$$\mathrm{F_{drag,water}} = \frac{1}{2} \mathrm{c_d} \rho_{water} A v^2 \tag{3}$$

(3) Buoyancy[Error! Reference source not found.]: When an object is immersed in a fluid, it will experience an upward buoyancy force equal to the weight of the object displacing the fluid. The buoyancy formula is:

$$F_b = \rho_{water} V \tag{4}$$

CONSTRUCTION OF THE RETRIEVAL RANGE OF SUBMERGED ITEMS BASED ON PHYSICAL MODELS

Analysis of the Problem

Since the direction of the wind speed is unknown, the direction of the force exerted on the object in the air is also unknown. However, the situation is the same or similar in each direction, so this article chooses one direction to use Python for graphing analysis, resulting in a displacement value that can be used as the search radius, as shown in the following Figure 1:

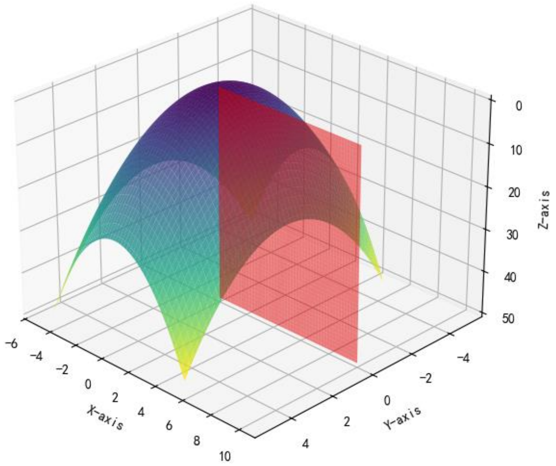

Figure 1. Choose a direction for the wind speed and water flow

The Establishment of Physical Model

Firstly, establishes the physical model of the object falling into the water**Error! Reference source not found.**, carries on the f orce analysis to the object falling, and establishes the object settlement movement track model, as shown in Figure 2:

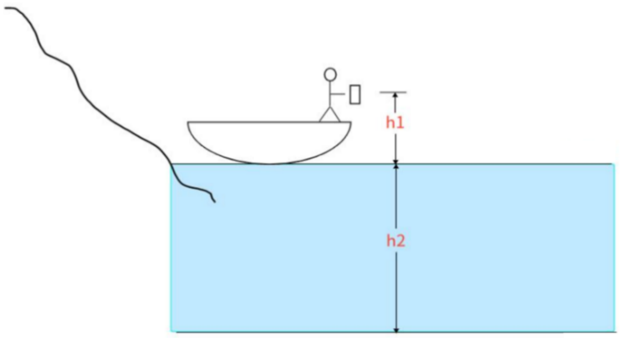

Figure 2. Object drop simulation

Motion in the air:

objects are affected by gravity and air resistance, as shown in Figure 3:

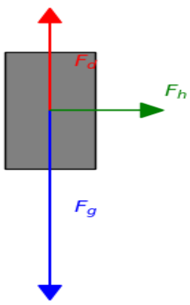

Figure 3. Force analysis of objects falling in the air

The differential equation model describing the motion of the object is constructed using formulas (1)-(4):

In the horizontal direction (taking wind speed into account):

$$m\frac{d_{v_z}}{d_t} = -c_d(v_x - v_{wind}) \tag{5}$$

m is the mass of the object, t is the time, c_d is the air resistance coefficient,v_x is the horizontal velocity, v_{wind} is the wind velocity.

According to Newton's second law, the force on an object is its mass times its acceleration: $F = ma$;

where the resultant force in the horizontal direction is primarily the effect of air resistance and wind speed on the motion of the object.

Therefore, the horizontal dynamic equation can be written as:

$$m\frac{d_{v_z}}{d_t} = -\frac{1}{2}c_d\rho_{air}Av_x^2 \tag{6}$$

Due to the presence of air resistance, the horizontal dynamic equation can be:

$$F_{drag,air} = \frac{1}{2}c_d\rho_{air}Av^2 \tag{7}$$

But in order to simplify the calculation in practice, the comprehensive influence of air resistance and wind speed is reduced to a simple expression, namely formula (5), which makes the calculation more convenient, direct expression of the wind speed on the horizontal movement of objects.

Speed at which horizontal direction can be obtained by sorting and integrating:

$$F_{drag,air} = \frac{1}{2}c_d\rho_{air}Av^2 \tag{8}$$

v_{0x} is the initial horizontal velocity. Vertically:

$$m\frac{d_{v_y}}{d_t} = -mg - c_d v_y \tag{9}$$

g is a gravitational acceleration, about $9.79 m/s^2$.

Organize and integrate to obtain vertical velocity:

$$v_y(t) = \left(v_{0y} + \frac{mg}{c_d}\right)e^{-\frac{c_d}{m}t} - \frac{mg}{c_d} \tag{10}$$

v_{0y} is the initial vertical square velocity.

Motion in water:

after an object falls into the water, it is affected by buoyancy, gravity, and water resistance, as shown in Figure 4:

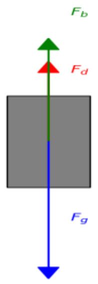

Figure 4. Force analysis of objects falling into the water

Horizontal:

$$m\frac{d_{v_x}}{d_t} = -kv_x \tag{11}$$

By sorting and integrating, the horizontal velocity formula is obtained as follows:

$$v_x(t) = v_{0x}e^{-\frac{k}{m}t} \tag{12}$$

Vertically:

$$m\frac{d_{v_y}}{d_t} = -mg + F_b - kv_y \tag{13}$$

Where buoyancy $F_b = \rho_{water}vg$,k is the water resistance coefficient,ρ_{water} is the water density, V is the volume of the object.

By sorting and integrating, the vertical velocity formula is obtained as follows:

$$v_y(t) = \left(v_{0_y} + \frac{F_b - mg}{k}\right) e^{-\frac{k}{m}t} - \frac{F_b - mg}{k} \quad (14)$$

SOLVING THE PHYSICAL MODEL

Mobile Phone Falls into the West Lake Area

Assuming that the heavy mobile phone falls into the water, here is an example of Mate60 in Huawei, China. The cross-sectional area, mass, length, width and height of the mobile phone can be determined by network verification data as shown in Figure 5:

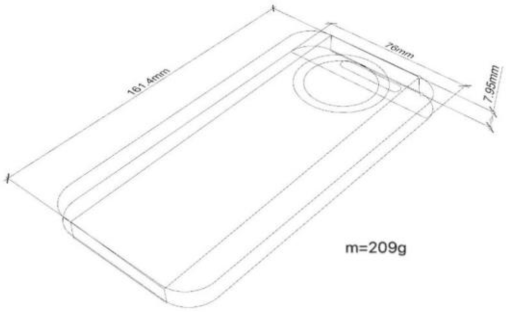

Figure 5. mobile phone model diagram

The wind speed and falling point, Use the formulas from the physical modeling process (5)-(14) to consider the parameters:

$g = 9.79 m/s^2$, air density $1.29 kg/m^3$, water density $1000 kg/m^3$, air resistance coefficient 0.47, water resistance coefficient 0.5, cell phone cross-sectional area $0.0006042 m^2$, with a mass of 0.209 kg and a wind speed of 0.5 m/s, the volume of the phone is calculated to be $0.00009752 m^3$.

According to the survey, the average height of Chinese men is 169.7 cm, while the average height of Chinese women is 158 cm, So suppose the height of the person is 1.7 m, that is, the height of the mobile phone falling is $h_1 = 1.7m$. Of course, here if you consider the actual drop height such as pocket height and other factors, you can adjust the drop start height h_1 for similar applications.

Substitute the parameters into the physical model and Python was used to carry out numerical simulation, resulting in a simulated picture of the phone's whereabouts, as shown in Figure 6:

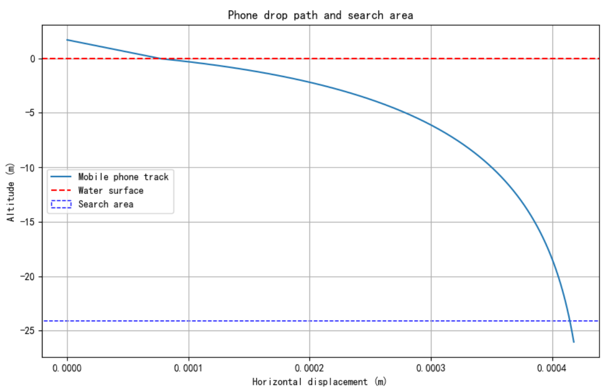

Figure 6. mobile phone falling into the water trajectory and search pattern

Results: suggested Search Range Center: (0.00042, -26.11) (the first number is horizontal offset, the second number is vertical offset); suggested search radius: 0.00042 m.

In lake water, when a mobile phone falls into the air, it is affected by the wind speed, which produces a velocity along the x-axis, which causes displacement along the wind direction. When the mobile phone falls into the water, it is no longer affected by the wind speed, under the influence of water resistance, the velocity in the x-axis is reduced to zero, and then it moves in a straight line with constant acceleration in the vertical direction of the water until it reaches the bottom.

It can be seen that the effect of wind speed on the landing of the mobile phone is very small, The lateral offset of the mobile phone is only 0.00042 m when the wind speed is 2 m/s, and when the wind direction is different, the offset of all other cases is 0.00042 m, this greatly narrows the search area.

Mobile Phone Falls into the Grand Canal

Contrast to the West Lake scenario, the Beijing-Hangzhou Grand Canal**Error! Reference source not found.** needs to take into account the i mpact of water flow on the phone's landing. The force generated by the flowing water can cause the phone to move, thereby changing its position and making the landing point different from the initial drop point.

air resistance:

$$F_{drag,air} = \frac{1}{2} c_d \rho_{air} A v^2 \tag{15}$$

ρ_{air}: coefficient of air resistance, A: Mobile phone windward area, V: Mobile phone movement speed.

Equation of motion in the horizontal direction:

$$m \frac{d_{v_x}}{d_t} = -\frac{1}{2} c_d \rho_{air} A (v_x - v_{wind})^{\ 2} \tag{16}$$

According to Newton's second law of motion, the force acting on an object is equal to its mass multiplied by its acceleration, i.e $F = m \cdot a$

In the horizontal direction, the main force acting on the object is air resistance. Therefore, the dynamic equation can be written as:

$$-F_{drag,air} = m \frac{d_{v_x}}{d_t} \tag{17}$$

Substituting the expression for air resistance into the equation of dynamics, we obtain formula;

This equation indicates that the primary factor influencing the change in an object's speed while moving through the air is air resistance. The magnitude of the air resistance directly proportional to the square of the object's speed relative to the air. Therefore, as the object's speed increases, the air resistance it encounters will increase significantly

Equation of motion in the vertical direction:

$$\frac{d_{v_y}}{d_t} = -mg - \frac{1}{2} c_d \rho_{air} A v_y^2 \tag{18}$$

These formulas utilize Newton's second law of motion $F = m \cdot a$,and take into account the effects of drag and buoyancy.

Sports in the water:

In the water, the phone is affected by gravity, water resistance, and buoyancy. Water resistance is similar to air resistance but has a different density, as shown in Figure 7:

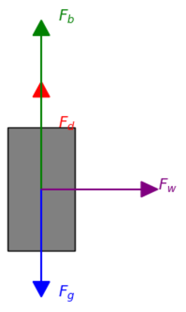

Figure 7. Force analysis diagram of a mobile phone falling into the Beijing-Hangzhou Grand Canal.

Formula for water resistance:

$$F_{drag,water} = \frac{1}{2} c_d \rho_{water} A v^2 \tag{19}$$

ρ_{water}: Water density.

Buoyancy can be expressed as:

$$F_{buoyancy} = \rho_{water} v_g \tag{20}$$

The equation of motion in the horizontal direction:

$$m \frac{d_{v_x}}{d_t} = -\frac{1}{2} c_d \rho_{water} A (v_x - v_{water})^2 \tag{21}$$

Equation of motion in the vertical direction:

$$m \frac{d_{v_y}}{d_t} = -mg + \rho_{water} v_g - \frac{1}{2} c_d \rho_{water} A v_y^2 \tag{22}$$

These formulas also utilize Newton's second law of motion, incorporating the effects of buoyancy and water resistance.

Model solving:

According to query-based computation, the parameter values can be determined:

$g = 9.79 m/s^2$, The air density is $1.29 kg/m^3$ and the water density is $1000 kg/m^3$, The resistance coefficient is 0.47, and the water resistance coefficient is 0.87, The cross-sectional area of the phone is $0.0006042 m^2$, and the mass of the phone is 0.209kg, which allows us to calculate the volume of phone as $0.00009752 m^3$,

Assuming the water flow speed is 1m/s and the wind speed is 0.5m/s, and the height of the person is 1.7m, then the height from which the phone falls is $h_1 = 1.7m$.

Substitute the parameters into the equation to get the simulation graph of the phone's fall, as shown in Figure 8:

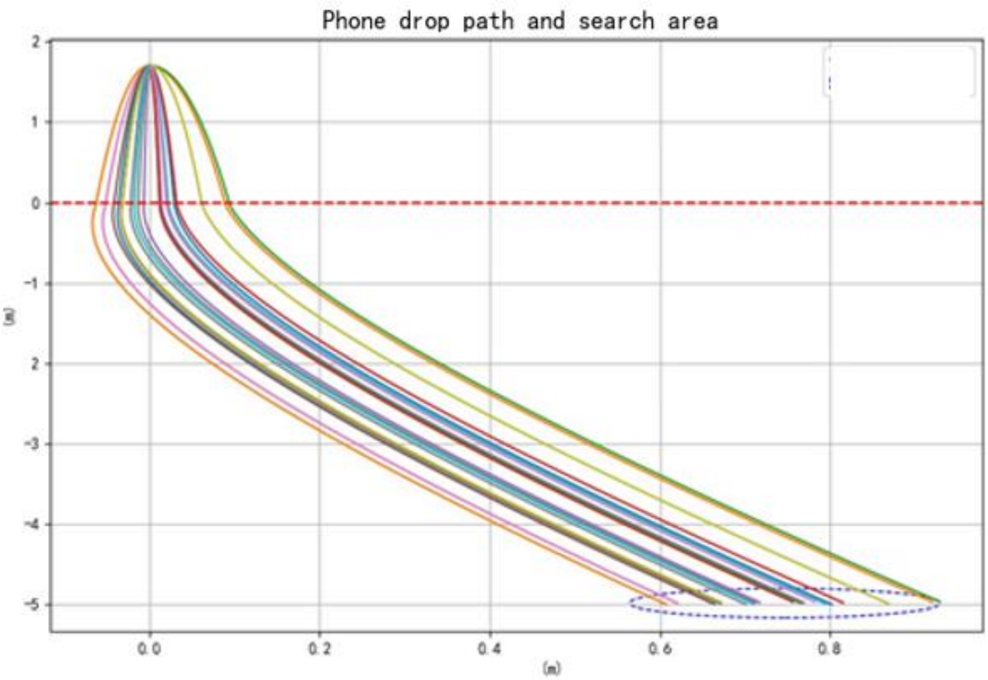

Figure 8. Simulation of the trajectory of a mobile phone falling into the Beijing-Hangzhou Grand Canal

After multiple simulations, Use the formulas from the physical modeling process (15)-(22), consider the parameters

Suggested search range center: $(0.75, -4.98)$

Suggested search radius： 0.18m

The falling phone is affected by the wind speed in the air, but due to its larger mass, the wind speed has a smaller impact on it resulting in a very small lateral displacement of the falling phone. After falling into the water, the phone is affected by the water flow, and the impact force generated the water flow causes the phone to move laterally in the water. Therefore, compared to still water, the Beijing-Hangzhou Grand Canal has a water flow, so the landing point of the phone will be away from the initial drop point of the tourist's phone.

Considering that the phone's own rotation may cause the not to move in the direction of the wind speed, the figure shows the phone moving along the negative x-axis as it falls through the air.

ID Card Falls into the West Lake Area

The standard ID card is 6.61 g in mass, 85.6 mm in length, 54 mm in width and 1 mm in thickness.The average density of the ID card is approximately $\rho_i = 1430 kg/m^3$, The density of the ID card is greater than that of water: $\rho_{water} = 1000 kg/m^3$,so the ID card will gradually sink into the water, and will not float.

The force analysis of the ID card during air landing is similar to Figure 4, $F_d$is the air resistance of the ID card during air free fall, F_g is the gravity force of the ID card, replace the cell phone information with the identity information into the equation to get the identity card fell into the lake water trajectory diagram, as shown in Figure 9:

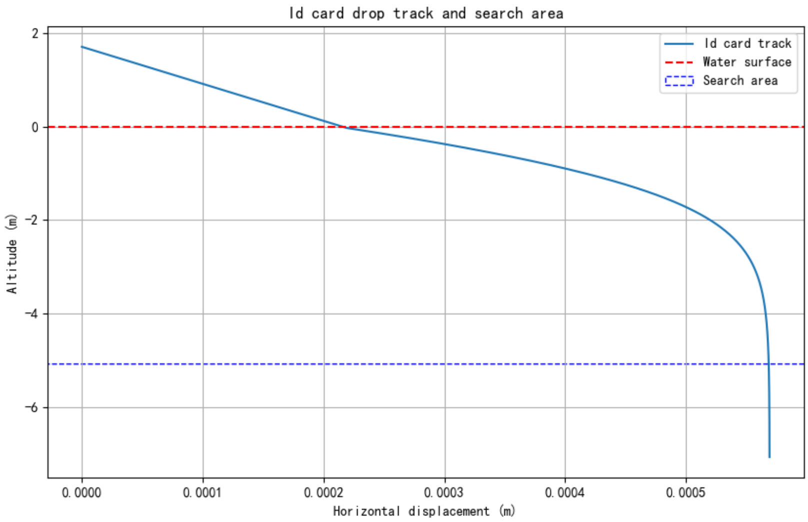

Figure 9. ID card into the water track and search area

Results: suggested Search Range Center: (0.00057, -7.07); suggested search radius: 2m.

Like a cell phone, a dropped ID is affected by wind speed in the air to produce an X-axis velocity, which slows to zero when it falls into the water, the effect of wind speed on the ID card is obviously greater than the mobile phone, so the horizontal offset of the ID card is larger than the horizontal offset of the mobile phone, the underwater range of about 2 meters can be found.

ID Card Falls into the Grand Canal

Due to the moving water of the Beijing-Hangzhou Grand Canal, which is different from the still water of the West Lake in Hangzhou, water flow is fast and turbulent. Moreover, the quality of the second-generation ID card is small, and its density is greater than that of water. Therefore, the ID card falls into the water, it will not only gradually sink but also move along the direction of the turbulent water flow.

By the same token, the card is affected by forces in the air, The force analysis diagram of the ID falling in the air over the Beijing-Hangzhou Grand Canal is similar to Figure 3.

Where F_d is the air resistance experienced by the ID card in free fall, and F_g is the gravitational force acting on the ID card itself.

The force analysis of an ID card falling into the Beijing-Hangzhou Grand Canal is similar to Figure 7.

WhereF_d is the air resistance encountered by the ID card in free fall in the air, F_g is the gravitational force acting on the ID itself, F_d is the resistance of water to the ID card, and F_w is the impact force of the water flow in the direction of movement on the card.

The rest of the parameters are the same as 4.3 substituting the information of the mobile phone with that of the ID card into the to obtain, as shown in figure 10:

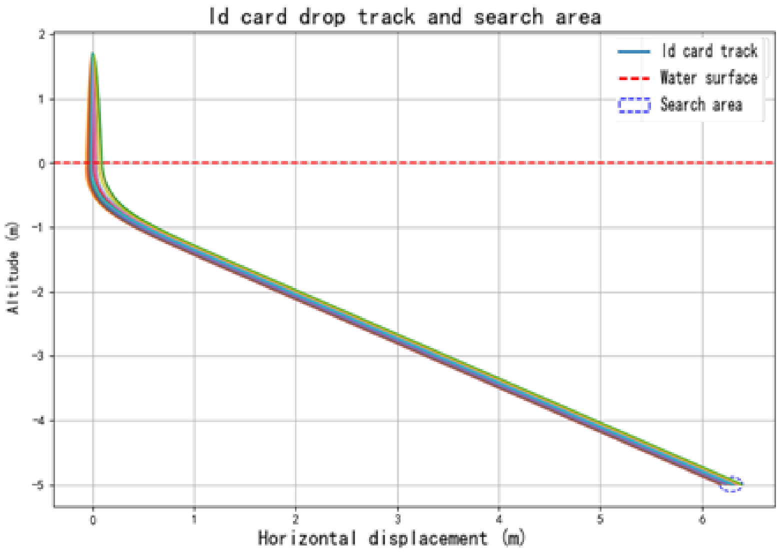

Figure 10. Trajectory map of the ID card falling into the Beijing-Hangzhou Grand Canal

The calculated results are as follows:

Suggested search range center: $(6.29, -5.00)$

Suggested search radius:11m

Similar to the scenario of the phone falling in problem two, the ID card falling in the air will experience a velocity along the x-axis due to influence of wind speed, which will reduce to 0 upon hitting the water; however, due to its smaller mass, the ID card is more significantly affected by speed than the phone, thus the lateral displacement of the ID card will be greater than that of the phone. Meanwhile, since the water in the Beijing-Hang Grand Canal is flowing water, the ID card will move with the rapid current, necessitating a larger search area.

CONCLUSION

According to the simulation results, the scientific fishing strategy is put forward. The effective search range is determined by analyzing the distribution of the falling points of different types of objects under the conditions of wind speed and water flow, thus the fishing strategy is optimized, and the fishing efficiency and success rate are improved. And the operation is simple, the parameters are clear, the calculation process is simple, by determining the center and radius of the scope, the salvage work is more efficient and convenient.

Python was used to simulate the movement of objects (such as mobile phones and ID cards) in water. Based on the simulation results, the significant influence of the velocity and direction of the water flow on the final falling point is determined, which provides the precise theoretical support for the salvage work.

The method adopted in this paper has clear parameters, improves the Operability of the method, enhances the predictability and reliability of the results, ensures the effectiveness of the method by clear thinking, and is easy to operate, the method has a high feasibility in practical application, and can be widely used not only in the salvage of falling objects, but also in the calculation of falling areas of other things, can Be More intuitive solution to the problem, easy to use and understand, conducive to the realization of procedures, intuitive and easy to understand, the use of value.

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